ON QUALITATIVE BEHAVIOUR OF A CLASS OF PIECEWISE-LINEAR CONTROL SYSTEMS. PART I. BASIC MODELS

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Key words: Complex systems, Hybrid control systems (HCS), Piecewise-linear (PL) systems, Qualitative models.

Specific modelling and simulation tools have recently emerged for the study of complex systems, allowing systematic prediction of their behaviour. In case of incomplete knowledge of the original physical system, qualitative modelling tools are preferred to quantitative ones. Starting from a *hybrid control systems* (HCS) framework in control engineering, this paper proposes a qualitative modelling approach for a class of *piecewise linear* (PL) differential systems, firstly introduced as a genetic network model and having a natural feedback structure. The thresholds limits imposed to the state variables partition the state space into open regions and the resulting qualitative model is a logical abstraction of the families of continuous trajectories mapped to this partition. Part I describes the basic general models and associated problems.

1. **INTRODUCTION AND MOTIVATION**

Systems with dynamics described by a set of continuous-time differential equations in conjunction with a discrete event process are usually referred to as hybrid systems. They can be viewed either as special classes of non-linear systems, with discontinuous vector fields, or as special classes of automata, thus admitting also a qualitative description [1].

Quantitative approaches of such combined dynamics require specification of numerical coefficients values in the differential system, which may not be exactly known for the corresponding physical system. In contrast, qualitative approaches, based on inequality-like restrictions imposed to the state variables, provide solutions in case of incomplete system knowledge [2] and have emerged, in the past decade, as efficient modelling and simulation tools in areas like systems biology [3] and control engineering [4], among others.

The basic model studied in this paper is a *piecewise-linear* (PL) differential system, whose right-hand side changes when the state variables reach given

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Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **54**, *1,* p. 95–104, Bucarest, 2009

threshold limits. If the state space is partitioned according to these limits, the evolution of continuous trajectories can be abstracted to a qualitative model, represented by a logic automaton: the discrete states correspond to partition cells in the continuous state space while state transitions occur when threshold limits are crossed.

The proposed qualitative modelling approach is based on a variant of a *hybrid control systems* (HCS) framework, firstly introduced by Antsaklis and his co-workers [5]. Under certain assumptions, the PL differential system can be abstracted to a *discrete event system* (DES), called DES-plant, without need to integrate the original state equations. The discrete evolution of the DES-plant model corresponds to the qualitative behaviour of the differential system. In addition, a switching control law is deduced and implemented in simulation programs, making possible a comparison between the qualitative evolutions and their numerical counterparts in the partitioned state space. The original PL state equations were firstly introduced in the genetic networks literature [6], and a different qualitative model was proposed for them in [3]. Nevertheless, the class of PL differential systems can be viewed also from a control engineering perspective, as a model of a generic nonlinear feedback system, with a supervisor driving a linear stable plant.

The paper is organized as follows. In Part I, the basic PL model is described in Chapter 2, followed by an overview of the HCS framework in Chapter 3. The core is the qualitative construction of the DES-plant abstraction, which describes the interaction between the continuous trajectories and the hypersurfaces of the state partition. Finally, the construction of a DES-plant automaton as a qualitative representation of a PL differential system is considered.

Part II is dedicated, without loss of generality, to the construction of a DES-plant abstraction for a particular second order PL differential system, followed by the comparative analysis of relevant qualitative and simulated evolutions.

2. A CLASS OF PL DIFFERENTIAL SYSTEMS

Consider the PL differential system in [3], [6] given by

$$\dot{\boldsymbol{x}} = \boldsymbol{v}(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{x})\boldsymbol{x}\,,\ \boldsymbol{x} \geq 0\,, \tag{1}$$

with $\boldsymbol{x} \in \mathbf{R}^n$ the state vector, $\boldsymbol{v} = (v_1, \ldots, v_n)'$, $\boldsymbol{g} = \mathrm{diag}(g_i)_{i=1:n}$. For each $i \in \overline{1:n}$, $v_i : \mathbf{R}^n_{\geq 0} \to \mathbf{R}_{\geq 0}$ is $v_i(\boldsymbol{x}) = \sum_{l \in L_i} k_{il} d_{il}(\boldsymbol{x})$, with $k_{il} > 0$, $d_{il} : \mathbf{R}^n_{\geq 0} \to \{0,1\}$ and L_i a possibly empty set of indices. The function $d_{il}(\boldsymbol{x})$ is a combination of step functions $s^{\pm}(x_i, \theta_i^j)$, with $s^{+}(x_i, \theta_i^j) = 0$, for $x_i < \theta_i^j$, $s^{+}(x_i, \theta_i^j) = 1$, for $x_i > \theta_i^j$, and $s^{-}(x_i, \theta_i^j) = 1 - s^{+}(x_i, \theta_i^j)$, where $\theta_i^j \geq 0$, $j \in \overline{1:J_i}$, are threshold values with $J_i \geq 1$. Usually $g_i(\boldsymbol{x}) = \alpha_i > 0$, $i = 1:n$. In [3] and [6], x_i, $i = 1:n$ have the significance of protein concentrations, with $v_i(\boldsymbol{x})$ and $g_i(\boldsymbol{x})x_i$ their rates of synthesis and degradation, respectively. In the sequel, the study is restricted to the case $L_i = 1$, $i = 1:n$, so the index l will be omitted.

From a control engineering perspective, equation (1) can be written as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x}), \tag{2}$$

with $\boldsymbol{A} = \text{diag}(-\alpha_i)_{i=1:n}$ the stable plant matrix and $\boldsymbol{u}(\boldsymbol{x}) \equiv \boldsymbol{v}(\boldsymbol{x})$ the nonlinear control law, depending on expressions $\text{sgn}(x_i - \theta_i^j)$, $i = 1:n$, $j = 1:J_i$.

3. A HCS FRAMEWORK AND THE DES-PLANT MODEL

In the HCS structure, the continuous plant is controlled, through an interface, by a discrete event controller (Fig.1a). Starting from a partition of the continuous state space, the plant coupled to the interface is abstracted to a DES, called DES-plant, and then the controller is built within the DES theory. The evolution of the DES-plant describes all possible *qualitative behaviours* of the plant, with initial states placed arbitrarily in partition cells. The structure of the HCS is firstly reviewed in brief, followed by the presentation of a technique for the qualitative construction of the DES-plant model. Details are given in [7], [8]. Finally, the conversion of the PL differential system to a DES-plant automaton is formulated as a distinct design problem in HCS framework.

3.1. THE STRUCTURE OF THE HCS

The continuous plant is modelled by the differential system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)), \tag{3}$$

where $\boldsymbol{x}(t) \in X \subseteq \mathbf{R}^n$ and $\boldsymbol{u}(t) \in U \subseteq \mathbf{R}^p$ are the state and control vector respectively, at time $t \in \mathbf{R}$. X is the continuous state space. The set of admissible control values $U = \{\boldsymbol{u}_1, \dots, \boldsymbol{u}_M\}$, with $M \geq 1$ is bijectively mapped to the *alphabet of control-symbols*

$$\widetilde{R} = \{r_1, \dots, r_M\} \tag{4}$$

by $\gamma : \widetilde{R} \to U$, $\gamma(r_m) = \boldsymbol{u}_m$, $m = 1:M$, such that the equivalence $U \sim \widetilde{R}$ holds.

The interface converts signals between the plant and the controller and it comprises *the actuator* and *the event generator. The actuator* converts a string of control-symbols to a vector of piecewise constant control-signals for the plant.

The state space partition is related to the system of threshold sensors, hence to *the event-generator*, which converts the plant's state trajectory $\boldsymbol{x}(\cdot)$, evolving in the partitioned state space, into a string of *plant-symbols*. Consider a set of $N \geq 1$ continuously differentiable indexed functionals

$$S_h^N = \{h_i : X \to \mathbf{R} \mid h_i \in C^1, i = 1:N\}, \tag{5}$$

defining the *partition* of the state space *X*.

The smooth hypersurfaces $\ker(h_i) = \{\boldsymbol{x} \in X \mid h_i(\boldsymbol{x}) = 0\}$ meet the nonsingularity conditions $\nabla_{\boldsymbol{a}}(h_i) \neq \boldsymbol{0}$, $\forall \boldsymbol{a} \in \ker(h_i)$, $i \in 1:N$, and each one separates the continuous state space into two disjoint half-spaces, $H_i^+ = \{\boldsymbol{x} \in X \mid h_i(\boldsymbol{x}) > 0\}$ and $H_i^- = \{\boldsymbol{x} \in X \mid h_i(\boldsymbol{x}) < 0\}$, respectively (Fig.1b).

Define $DX = X \mid Fr$, $Fr = \cup_{i=1}^{N} \ker(h_i)$. The *equivalence relation* induced by S_h^N is $rel \subset DX \times DX$, with $(\boldsymbol{x}_a, \boldsymbol{x}_b) \in rel \Leftrightarrow h_i(\boldsymbol{x}_a) h_i(\boldsymbol{x}_b) > 0$, $\forall h_i \in S_h^N$.

The *cellular space* $DX_{/rel} = C$ results from the intersection of hypersurfaces $\ker(h_i)$, $i = 1:N$, as a set of $\widehat{Q} \leq 2^N$ disjoint open cells, each cell being uniquely labelled by a symbol from the plant's *alphabet of discrete states*

$$\widetilde{P} = \{p_1, \ldots, p_{\widehat{Q}}\}. \tag{6}$$

The vector valued *quality function* $\boldsymbol{b}: X \to \{-1,0,1\}^N$ is defined by

$$\boldsymbol{b}(\boldsymbol{x}) = (\operatorname{sgn}(h_1(\boldsymbol{x})) \quad \ldots \quad \operatorname{sgn}(h_N(\boldsymbol{x})))'. \tag{7}$$

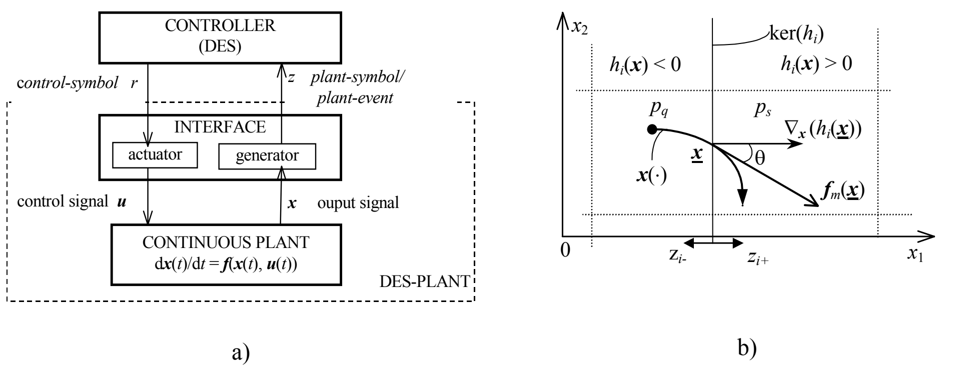


Fig. 1 – a) The architecture of a HCS; b) the occurrence of a plant-event (*i*+).

The value $\boldsymbol{b}(\boldsymbol{x})$, $\boldsymbol{x} \in X$, is *consistent* if and only if $\mathrm{sgn}(h_i(\boldsymbol{x})) \neq 0$, $\forall h_i \in S_h^N$ and *inconsistent* if else [7]. Denote $c_q \in C$ the cell labelled by $p_q \in \widetilde{P}$. Within each cell c_q, which is an equivalence class of *rel*, the functionals h_i, $i \in \overline{1:N}$, preserve their sign, so for $q = 1:\hat{Q}$, the *qualitative state* of p_q, given by $\boldsymbol{b}(\boldsymbol{x}) = \boldsymbol{b}_q$, $\forall \boldsymbol{x} \in c_q$, is well defined. The set of consistent qualitative values, or *qualitative plant states*, is

$$B = \{\boldsymbol{b}_1, \ldots, \boldsymbol{b}_{\hat{Q}}\}, \tag{8}$$

with $C \sim \widetilde{P} \sim B$.

The discrete states $p_q, p_s \in \widetilde{P}$ are *adjacent* if $\exists h_i \in S_h^N$ so that the vectors $\boldsymbol{b}_q, \boldsymbol{b}_s \in B$ satisfy the relations: $b_q^i b_s^i = -1$, $b_q^j b_s^j = 1$, $\forall j \in \overline{1:N}, j \neq i$. The *adjacency boundary* between two states $p_q, p_s \in \widetilde{P}$, separated by $\ker(h_i)$, is

$$A(h_i, q, s) = \{\boldsymbol{x} \in X \mid \mathrm{sgn}(h_i(\boldsymbol{x})) = 0, \mathrm{sgn}(h_j(\boldsymbol{x})) = b_q^j,\ j \neq i, j \in 1:N\}, \tag{9}$$

with $A(h_i, q, s) = A(h_i, s, q) \subseteq \ker(h_i)$.

A *plant-event* denoted (i+) or (i–), $i \in 1:N$, occurs whenever the continuous trajectory $\boldsymbol{x}(\cdot)$ crosses the hypersurface $\ker(h_i)$, $h_i \in S_h^N$, in the positive or negative direction, respectively. A *sufficient condition* for the occurrence of a plant-event (i+) at the time $t_e \in \mathbf{R}$ is

$$h_i(\boldsymbol{x}(t_e)) = 0 \ \wedge\ \dot{h}_i(\boldsymbol{x}(t_e)) > 0, \tag{10}$$

and similarly for the plant-event (i–). The *alphabet of plant-symbols* is

$$\widetilde{Z} = \{z_{1+}, z_{1-}, \ldots, z_{N+}, z_{N-}\} \cup \{\varepsilon\}, \tag{11}$$

where ε is the *silent event* and the plant-symbol z_{i+} / z_{i-}, $i \in \overline{1:N}$, is sent through the generator whenever the associated plant-event (i+)/(i–) occurs (Fig.1b).

The DES-plant model is the automaton $G_p = \{\widetilde{P}, \widetilde{R}, f_p, \widetilde{Z}, g_p\}$, where $\widetilde{P}$ is the set of discrete states, $\widetilde{R}$ is the *input* alphabet of control-symbols, $\widetilde{Z}$ is the output alphabet of plant-symbols, $f_p : \widetilde{P} \times \widetilde{R} \to 2^{\widetilde{P}}$ is the state transition function and $g_p : \widetilde{P} \times \widetilde{P} \to \widetilde{Z}$ is the output function. The dynamical equations are

$$p(k+1) \in f_p(p(k), r(k)), \; g_p(p(k), p(k+1)) = z(k+1), \tag{12}$$

where $p(k), p(k+1) \in \widetilde{P}$, $z(k+1) \in \widetilde{Z}$, $r(k) \in \widetilde{R}$, $\forall k \geq 0$ [8].

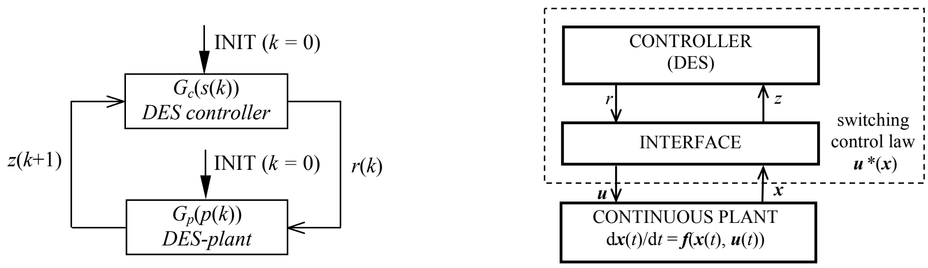
The DES controller is a Moore machine $G_c = \{\widetilde{S}, \widetilde{Z}, f_c, s_0, \widetilde{R}, g_c\}$, where $\widetilde{S}$ is the finite set of discrete states, $s_0 \in \widetilde{S}$ is the initial state, $\widetilde{Z}$ is the input alphabet, $\widetilde{R}$ is the output alphabet, $f_c : \widetilde{S} \times \widetilde{Z} \to \widetilde{S}$ is the state transition function and $g_c : \widetilde{S} \to \widetilde{R}$ is the output function. The dynamical equations [8] are

$$\begin{aligned} f_c(s(k), z(k+1)) &= s(k+1), \; s(0) = s_0, \\ g_c(s(k+1)) &= r(k+1), \; g_c(s(0)) = r(0), \end{aligned} \tag{13}$$

with $s(k), s(k+1) \in \tilde{S}$, $z(k+1) \in \widetilde{Z}$, $r(k) \in \widetilde{R}$, $\forall k \geq 0$.

As the initial state of the differential system (3) is known only to reside within a partition cell, a state transition in the DES-plant automaton corresponds to the behaviour of a *family* of controlled continuous trajectories, so G_p may be nondeterministic, while G_c is deterministic. Consequently, there are two modelling approaches for the HCS model: a pure logical one, where the control part is an automaton (Fig.2a) and a classical nonlinear one (Fig. 2b). In the latter, DES controller and interface play the role of a *switching control law*, which, in the absence of hysterezis, can be written in the form

$$\boldsymbol{u}^*(\boldsymbol{x}) = F(\operatorname{sgn}(h_1(\boldsymbol{x})), \ldots, \operatorname{sgn}(h_N(\boldsymbol{x}))), \; F : \mathbf{R}^N \to \mathbf{R}^p. \tag{14}$$



a) b)

Fig. 2 – The modelling approaches of the HCS: a) a logical supervision system; b) a nonlinear control system.

3.2. THREE BASIC PROBLEMS OF THE HCS FRAMEWORK

There are three main design problems concerning the HCS framework: firstly the construction of the DES-plant automaton, secondly the synthesis of the DES controller, performed within the logical approach (Fig. 2a) for a desired discrete evolution and, finally, the extraction of the switching control law for the continuous plant, corresponding to the already designed DES controller (Fig. 2b).

More formally, denote in (3)

$$\boldsymbol{f}_m(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}_m),\ \forall \boldsymbol{u}_m \in U. \tag{15}$$

Problem 1: Construction of the DES-plant automaton $G_p = \{\widetilde{P}, \widetilde{R}, f_p, \widetilde{Z}, g_p\}$.

Given: (*i*) the set of control inputs and the alphabet of control-symbols $U \sim \widetilde{R}$ (4), (*ii*) the state space partition given by $\ker(h_i)$, $h_i \in S_h^N$ (5) (*iii*) the equations of the controlled plant $\dot{\boldsymbol{x}} = \boldsymbol{f}_m(\boldsymbol{x})$, with $\boldsymbol{x}_0 \in c_q \in C$, for $q = 1:\hat{Q}$, $m = 1:M$ and defining (*iv*) the alphabet of discrete states and the cellular space $\widetilde{P} \sim C$ (6), (*v*) the set of qualitative states *B* (8) and (*vi*) the alphabet of plant-symbols $\widetilde{Z}$ (11), *construct* the functions f_p and g_p.

There is no general construction method, but for second order systems, a direct solution is provided by the phase portrait of the plant, mapped to the state space partition.

Problem 2 (simplified version): Synthesis of the DES controller G_c.

Given a desired path* $\omega_p = p(0), p(1), \ldots, p(l)$ in G_p, find a control sequence $\omega_r = r(0), r(1), \ldots, r(j-1) \in \widetilde{R}^*$ with $j \in \overline{0, l-1}$ maximal, such that the *controlled path* $p(0) \xrightarrow{r(0)} p(1) \xrightarrow{r(1)} \ldots \xrightarrow{r(j-1)} p(j)$ is deterministic.

The second problem is essentially an optimization problem and it may have no solution, a single solution or several solutions. The state transition function f_c and the output function g_c of the Moore machine G_c result from the control sequence $\omega_r \in \widetilde{R}^*$ and from the sequence of plant-symbols $\omega_z \in \widetilde{Z}^*$ associated to the deterministic controlled path in G_p. An extended version of *Problem* 2, implying a desired behaviour in the DES-plant defined as a sub-automaton together with a solution within the DES control theory are proposed in [9].

Problem 3: Synthesis of the switching control law $\boldsymbol{u}^*(\boldsymbol{x})$.

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* Intuitively, a state $p_b \in \widetilde{P}$ is reachable from $p_a \in \widetilde{P}$ if there exists a path in G_p, which starts in p_a and ends in p_b. Solving *Problem* 1 answers also the reachability problem.

Given a deterministically controlled path $p(0) \xrightarrow{r(0)} p(1) \xrightarrow{r(1)} \ldots \xrightarrow{r(j-1)} p(j)$, find the associated switching control law $\boldsymbol{u}^*(\boldsymbol{x})$, such that the relation $\boldsymbol{u}^*(\boldsymbol{x}) = \gamma(r(k))$ is satisfied, $\forall \boldsymbol{x} \in c(k) \in C$, with $c(k)$ the cell labelled with $p(k)$, $k = 0 : j$. ◊

There is no general solving method and a heuristic solution, in the case without hysterezis, is presented in [7].

3.3. THE QUALITATIVE CONSTRUCTION OF THE DES-PLANT MODEL

The solution to the previously formulated *Problem* 1 is based on the information about all families of controlled continuous trajectories, parameterised by initial states located in the partition cells, respectively. This may be a challenge from both analytical and numerical viewpoint and hence it is important to consider approaches making possible the DES-plant construction without integrating the differential system (3).

Formally, the construction of G_p requires information about the trajectories of the differential systems $\dot{\boldsymbol{x}} = \boldsymbol{f}_m(\boldsymbol{x})$, $\forall \boldsymbol{x}_0 \in c_q \in C$, $q = 1 : \hat{Q}$, $m = 1 : M$.

Consider the following assumptions.

A0. In the HCS, the plant events do *not* occur simultaneously.

A1. *Absence of inflexion points at the crossing moments*.

Given $\forall h_i \in S_h^N$, $\forall m \in \overline{1:M}$ and $\forall \boldsymbol{x}_0 \in X$, with $\varphi_m(\cdot)$ the solution of $\dot{\boldsymbol{x}} = \boldsymbol{f}_m(\boldsymbol{x})$, $\boldsymbol{x}(0) = \boldsymbol{x}_0$, the function $h_i \circ \varphi_m : \Im \to \mathbf{R}$, with $\Im \subseteq \mathbf{R}$ interval, has the property: if $\exists t_e \in \Im \subseteq \mathbf{R}$ with $h_i(\varphi_m(t_e)) = 0$ and $[\mathrm{d}h_i(\varphi_m(t)) / \mathrm{d}t]_{t=t_e} = 0$, then t_e is *not* an inflexion point.

A2. *Constant trajectories orientation on the crossing border*.

For any $p_q, p_s \in \widetilde{P}$ and $\forall m \in \overline{1:M}$, if p_q and p_s are adjacent on $\ker(h_i)$, $h_i \in S_h^N$, then, $\mathrm{sgn}(< \boldsymbol{f}_m(\boldsymbol{x}), \nabla_x h_i >)$ is constant for $\forall \boldsymbol{x} \in A(h_i, q, s)$.

A3. *Absence of undetectable continuous evolutions*.

For any $c_q \in C$ and any $m \in \overline{1:M}$, the *family* of solutions of $\dot{\boldsymbol{x}} = \boldsymbol{f}_m(\boldsymbol{x})$, parameterised by $\boldsymbol{x}(0) = \boldsymbol{x}_0 \in c_q$, either crosses the border of c_q at some $t_e > 0$, or it remains within the cell c_q.

Consequence of **A0**: the plant's trajectories do not pass through the intersection of two or several hypersurfaces of the partition (5).

Assuming that **A0** is true, a plant-event (i+) occurs if and only if $\exists \underline{\boldsymbol{x}} \in \ker(h_i)$ and $r_m \in \widetilde{R}$ s.t. $\boldsymbol{x}(t_e) = \underline{\boldsymbol{x}}$, $\boldsymbol{u}(t) = \gamma(r_m)$, $t \in (t_e^-, t_e^+)$ and $\boldsymbol{f}_m^{'}(\underline{\boldsymbol{x}}) \cdot \nabla_{\underline{\boldsymbol{x}}} h_i > 0$, as illustrated in Fig.1b (and similarly, with adequate changes, for (i–)) [7]. Recall that in (10) $\dot{h}_i(\boldsymbol{x}(t)) = < \nabla_{\boldsymbol{x}(t)} h_i, \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) >$.

CRITERION (a qualitative solution to Problem 1). *Consider that* **A0**-**A3** *are true.* A) *Given* $p_q, p_s \in \widetilde{P}$ *and* $r_m \in \widetilde{R}$ *in* G_p, *the state transition* $p_s \in f_p(p_q, r_m)$ *is true if and only if* p_q, p_s *are adjacent on some* $\ker(h_i)$, $h_i \in S_h^N$ *and*

$$b_q^i(\boldsymbol{f}_m^{'}(\boldsymbol{x}) \cdot \nabla_x h_i) < 0, \ \forall \boldsymbol{x} \in A(h_i, q, s), \tag{16}$$

with b_q^i *is the i-th component of* $\boldsymbol{b}_q \in B$. B) *If* $p_s \in f_p(p_q, r_m)$, *then* $g_p(p_q, p_s) = z_{i+} \in \widetilde{Z}$ *if and only if* $b_q^i = -1$ *(and similarly* $b_q^i = 1$, *for* $z_{i-} \in \widetilde{Z}$ *).*◊

The proof is given in [7].

Based on the *Criterion*, the DES-plant automaton can be constructed *without integrating* the equations (3). The price paid for this simplification is given by the restriction introduced by previous assumptions **A0**-**A3**, which can be tested for their validity only in particular and reduced order cases. Note that in any case, (10) and (16), respectively are sufficient conditions for a state transition occurrence.

3.4. FROM A PL DIFFERENTIAL SYSTEM TO A CONTROLLED DES-PLANT AUTOMATON – PROBLEM STATEMENT

The problem of interest is essentially the characterization of qualitative behaviours of the PL differential system given by equations (1) or (2), based on the previously described HCS framework. More specifically, starting from a plant controlled by a switching control law, the goal is to construct the associated DES-plant automaton, without disposing of an explicitly defined state space partition. The resulting automaton corresponds to the control scheme in Fig.2a, hence it generates the closed loop qualitative behaviours.

Problem 4: Construction of DES-plant model of a PL differential system.

Given: (i) the state space $X \subseteq \mathbf{R}^n$ and, for each $i \in \overline{1:n}$, the J_i threshold limits $\theta_i^j \geq 0$, $j = 1 : J_i$, together with (ii) the state equations in (2), rewritten in the form

$$\dot{x}_i = -\alpha_i x_i + u_i(\boldsymbol{x}), \ \alpha_i > 0, \ i = 1 : n, \tag{17}$$

where $\boldsymbol{x} \in X$ is the state and the control is a nonlinear function of state variables

$$u_i(\boldsymbol{x}) = k_i F_i(\mathrm{sgn}(x_1 - \theta_1^{j_1}), \ldots, \mathrm{sgn}(x_n - \theta_n^{j_n})), \quad (18)$$

with $k_i > 0$, $j_i \in \overline{1:J_i}$ and $F_i : \{-1,0,1\}^n \to \mathbf{R}$, $i = 1:N$, *construct* the input data (*i*)-(*vi*) for *Problem* 1 and *solve* the corresponding *Problem* 1.

The solution to *Problem* 4 will be discussed within a case study in the second part of the paper, followed by general conclusions.

Received on August 25, 2008

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