REDUNDANCY OPTIMAL ALLOCATION FOR SERIES – PARALLEL SYSTEMS APPLIED TO THERMAL POWER PLANTS

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The paper is presenting a model of optimal allocation of redundancy for auxiliaries belonging to turbo generator groups. They are considered as series-parallel systems. Optimization criterion that has been chosen aims the minimization of acquisition and installation costs, exploitation costs and all other costs due to non-operation. In order to solve the model two algorithms were developed: one is heuristic and the other one is metaheuristic, based on Ant Colony Optimization Algorithm. A case study is presented at the end of the article, showing the methodology for a 55 MVA coal-fired turbo generator group.

1. INTRODUCTION

Thermal power plants are important structures of the power systems, due to very high investment costs. Also, different consequences might occur at suppliers and/or customers after an equipment failure. The configuration of thermal power plants must cover the requirements of the supplied electric power quantity and quality. Consequently, to assign the optimal configuration of thermal power plants, mathematical models must include technical aspects connected to investments and reliability.

The main objective of the reliability optimization is maximizing the reliability-cost ratio, developing, mostly, two directions, as follows:

- maximizing the system reliability with respect of some costs constraints;
- minimizing the system cost, but insuring a minimal reliability level.

The redundancy allocation problem has previously been analyzed for many different system structures, objective functions and time-to-failure distributions. Generally, the problem domain has been limited to series-parallel systems with active redundancy or “k-out-of-n” systems consisting of a single subsystem.
There has been significant research activity devoted to different forms of the redundancy optimization problem. A comprehensive overview of system reliability optimization problem formulations and solution methodologies is provided in [1]. For a single subsystem, there have been research findings describing optimal configurations for the “k-out-of-n” subsystem. The reliability optimization and the redundancy allocation problem for the “k-out-of-n” systems have been analyzed using different optimization algorithms [6, 7, 8].

For multiple series subsystems, the problem has been formulated to maximize system reliability and solved using dynamic programming [2], integer programming [3], heuristic algorithms [9, 10].

Finding the optimal structure of a thermal power plant is an allocation problem that might be approached as combined optimization problem. In order to solve this type of problem, notable results had been achieved applying metaheuristic methods, such as Genetic Algorithms [4, 5, 15], Simulated Annealing [11, 17], Tabu Search [18], Ant Algorithms [12, 13, 16], Particle Swarm Optimization [14] and others. The above mentioned algorithms have also been used to solve different kind optimization problems of power plants: planning and scheduling short term power generation in thermal power plants [16, 19], unit commitment [15, 17, 18, 19], hydroelectric generation scheduling [21] and power plant maintenance scheduling optimization [20].

This paper aims to develop a mathematical model based on costs that will allow determining the optimal allocation for the equipment of auxiliaries that serve a turbo generator group. The assembly will be modeled as a series-parallel system. To solve this problem of optimal allocation, two algorithms were developed and applied: a heuristic one (HA) and a metaheuristic one (ACO), based on ant colony algorithm. The model was applied to a 55 MVA turbo generator group, having a typical structure for coal-fired thermal power plants.

2. OPTIMAL REDUNDANCY ALLOCATION PROBLEM

Power plant turbo generator units are served by several auxiliary equipment: coal mills for crushing the coal, air fans and burned-gas ventilators, feeding pumps for the steam generator (boiler), Bagger pumps for exhausting ash and boiler slag etc. Given both the important strains on these elements, the problems caused by their failure and the impact on the availability of the turbo generator unit, a certain redundancy is required in order to maintain the unit operation regardless the failure of auxiliaries. Generally, this auxiliaries subsystem that serve the turbo generator unit has different type of sizes: “2 × 50%”, “3 × 50%”, “1 × 100%”, “2 × 100%”, “n + 1”, “n + 1 + 1” etc. It can be seen that an exceedingly high reservation is used for some of these subsystems. From a technical and economical point of view, this is not at all justified. In the mean time, other subsystems have neither back-up, nor
a sufficient level of redundancy. Therefore, to have an optimal allocation of redundancy for unit auxiliaries we suggest modeling them as a series-parallel system.

A series-parallel system (Fig. 1) consists of \(s\) serial subsystems, each subsystem having \(n_i\) elements in operation and \(k_i\) elements in stand-by.

![Series-parallel system diagram](image)

**Fig. 1 – Series – parallel system (series “\(n_i + k_i\)” system: \(n_i\) working elements, \(k_i\) in stand-by).**

The aim of mathematical model is to establish the optimal allocation of the redundancy \((k_{1\text{ opt}}, k_{2\text{ opt}}, \ldots, k_{s\text{ opt}})\) for the series – parallel system. For this, as an optimization criterion, we propose to minimize the following objective function:

\[
\min F(k_1, k_2, \ldots, k_s) = \min \left\{ I(k_1, k_2, \ldots, k_s) + \left[ C(k_1, k_2, \ldots, k_s) + D(k_1, k_2, \ldots, k_s) \right] \cdot \sum_{i=1}^{s} (1+a)^i \right\},
\]

where: \(I(k_1, k_2, \ldots, k_s)\) – investment in the elements of the system [MU]. We consider that the investment is made in the first year of the study period; \(C(k_1, k_2, \ldots, k_s)\) – annual exploitation expenses for the system elements [MU/year]; \(D(k_1, k_2, \ldots, k_s)\) – annual damages caused by the system unavailability [MU/year]; \(T\) – study period; \(a\) – discounted cash flow; MU – monetary units [EUR, USD, RON etc.].

For the energy engineering field, the following values are generally accepted: \(T = 20\) years and \(a = 0.1\).

The above mentioned terms of the objective function are calculated using the following relations:

- **Investments:**
  \[
  I(k_1, k_2, \ldots, k_s) = \sum_{i=1}^{s} (n_i + k_i) \cdot C_{\text{inv}},
  \]
where $C_{ai}$ is the acquisition and assembly cost for the $i$-th element, including the auxiliary installation costs according to technological scheme [MU].

- Annual exploitation expenses:

$$C(k_1, k_2, \ldots, k_s) = \sum_{i=1}^{s} c_i \cdot (n_i + k_i) \cdot C_{ai},$$

(3)

where $c_i$ is the part of the acquisition and assembly cost representing the annual exploitation expenses, specific for the $i$-th element.

- Annual damages caused by the unavailability of the system: we suppose that the analytical expressions of the damage characteristics have linear variations of damage with respect of failure period, therefore we can write:

$$D(k_1, k_2, \ldots, k_s) = \sum_{j=1}^{s} \sum_{i=1}^{s} n_{d_{k_i+j}} \cdot (d_{t_{ij}} \cdot T_{d_{k_i+j}} + d_{a_{ij}}),$$

(4)

where: $d_{t_{ij}}$ – specific damage, direct proportional with the outage period of "$k_i+j$" elements [MU/h];

$d_{a_{ij}}$ – specific damage, direct proportional with the number of outages that occur when "$k_i+j$" elements are not running [MU/failure];

$n_{d_{k_i+j}}$ – number of annual failures (number of the system transitions into the state characterized by "$k_i+j$" damaged elements) [failures/year];

$T_{d_{k_i+j}}$ – probable failure time (occupancy time of the state which has "$k_i+j$" damaged elements) [h/failure].

Based on binomial modeling, annual number of transitions into the state having "$k_i+j$" damaged elements is:

$$n_{d_{k_i+j}} = \frac{(n_i + k_i)!}{(n_i - j)! \cdot (k_i + j - 1)!} \cdot \frac{\lambda_i^{k_i+j} \cdot \mu_i^{n_i-j-1}}{(\lambda_i + \mu_i)^{n_i}} \cdot 8760$$

(5)

where: $\lambda_i$ – failure rate of the $i$-th element [1/h]; $\mu_i$ – repairing rate of the $i$-th element [1/h].

Probable occupancy time of the state that has "$k_i+j$" damaged elements is the following:

$$T_{d_{k_i+j}} = \frac{1}{(k_i + j) \cdot \mu_i}.$$  

(6)

3. OPTIMIZATION ALGORITHMS

To find the optimal solution for problem (1) we developed two algorithms – a heuristic one (HA) and a metaheuristic (ACO) one – that are presented below.
They have been implemented in MathCAD program. The two calculation modules had been used in the case study presented in chapter 4.

3.1. HEURISTIC ALGORITHM

The developed heuristic algorithm (HA) is based on the procedure of direct investigation of the possible solutions, without being necessary to enumerate all of them. We wish to determine the optimal allocation \( OPTA = [k_{1,\text{opt}}, k_{2,\text{opt}}, \ldots, k_{s,\text{opt}}] \) for problem (1). The steps of this algorithm are the following:

**Step 1:** Enter values for: \( T, a, C_{ai}, c_i, \lambda_i, \mu_i, d_{ij}, d_{n,ij}, i = 1, s, j = 1, n_i \)

Establish first current allocation \( CA = [0 \ 0 \ldots 0] \) and calculate \( F(0, 0, \ldots, 0) \);

Establish \( OPTF = F(0, 0, \ldots, 0) \) and optimal allocation \( OPTA = CA \);

Generate the set of elementary allocations:

\[
EA_1 = [1 \ 0 \ldots 0]; \ EA_2 = [0 \ 1 \ldots 0]; \ldots; EA_s = [0 \ 0 \ldots 1];
\]

**Step 2:** For \( i = 1 \) to \( s \) establish the tested allocation: \( TA_i = CA + EA_i \);

Calculate \( F(k_1, k_2, \ldots, k_s) \), where \( (k_1, k_2, \ldots, k_s) \) are the elements of matrix \( TA_i \);

If \( F(k_1, k_2, \ldots, k_s) \leq OPTF \) then establish \( OPTF = F(k_1, k_2, \ldots, k_s) \) and optimal allocation \( OPTA = TA_i \); if not, than \( OPTF \) and \( OPTA \) are kept unchanged.

**Step 3:** If \( CA \neq OPTA \) then establish \( CA = OPTA \) and go to step 2.

If \( CA = OPTA \) the algorithm stops. \((OPTA, OPTF)\) is the optimal solution.

3.2. ANT COLONY OPTIMIZATION ALGORITHM

Ant colony optimization algorithm (ACO) is based on the similarity with the behavior of real ants that are capable of finding the shortest way between their nest and the source of food. In the process of seeking the food ants are laying down a certain substance, called pheromone, on all the paths they are traveling. As more and more ants transit the path, the amount of pheromone on this path increases and, therefore, the possibility that the path becomes a solution is higher, too. In the same time, the paths randomly chosen by ants in their wandering for food, that don’t lead to the source of food, lose their pheromones, and the probability of their selection decreases. Pheromones density laid down on a path is a communication guide between ants in order to choose the shortest path to food.

To apply the ACO algorithm for the submitted optimization problem – relation (1) – the following steps must be taken: solution representation, setting the occupancy rules of a state and the rules of pheromones updating, as well as presenting the main steps for implementing the algorithm.

a) *Representing the solution.* To determine the optimal configuration of the equipment that serve the turbo generator unit means to solve an allocation problem that has as a criterion the cost function \( F(k_1, k_2, \ldots, k_s) \), given by relation (1). Because the structure of the analyzed system includes \( s \) subsystems, the solution of the problem will be an allocation vector \( k = (k_1, k_2, \ldots, k_s) \), where its components \( k_i \) are
the number of elements allocated to each subsystem. Any \( k_i \) variable can take values between zero and the maximum admissible limit (\( k_{\max} \)), which is considered the same for all subsystems. The problem can be modeled considering that the discrete space of all solutions is figured by the cells of a matrix \( M \) having \((k_{\max} + 1) \times s \) dimension. A solution \( k = (k_1, k_2, \ldots, k_s) \) may be obtained when an ant is traveling through the cells of matrix \( M \), starting from a random position in column 1 to a random position in the last column \( s \), in accordance with the occupancy probability of \((j,i)\) state (where \( j = 0 \ldots k_{\max}, i = 1 \ldots s \)).

b) The rule of state occupancy. The occupancy probability \( p^{(h)}(j,i) \) of \((j,i)\) state by an ant \( h \) is calculated using the following relation:

\[ p^{(h)}(j,i) = \frac{\left(\tau(j,i)\right)^\alpha \cdot \left(\eta(j,i)\right)^\beta}{\sum_{r=1}^{x_{\max}} \left(\tau(j,r)\right)^\alpha \cdot \left(\eta(j,r)\right)^\beta}, \]

where: \( \tau(j,i) \) is the amount of pheromones laid down on the \((j,i)\) element of \( M \); \( \alpha, \beta \) are the parameters that indicate the importance of \( \tau(j,i) \), respectively \( \eta(j,i) \); \( \eta(j,i) \) gives additional heuristic information for a more rapidly calculation of the solution. The authors propose the following relation:

\[ \eta(j,i) = \frac{1}{k_{i,\text{opt}} - j + |\varepsilon|}, \]

where: \( k_{i,\text{opt}} \) is the best solution determined up to this iterative step; \( \varepsilon \) is a parameter experimentally developed by the authors, for the specific data of this problem it has optimal values in the range between \([0.25, 0.5]\).

c) Rule of pheromones updating. The amount of pheromone \( \tau(j,i) \) deposited on positions that aren’t on ant’s track have been decremented with the evaporation rate \( \rho \), using the following relation:

\[ \tau(j,i) \leftarrow (1 - \rho) \cdot \tau(j,i). \]

In the same time, the pheromones laid down on positions that belong to the best itinerary got at a certain iteration, are updated using the relation:

\[ \tau(j,i) \leftarrow \tau(j,i) + \frac{1}{\sigma \cdot F(k_{\text{opt}}^{\text{opt}})}. \]

The constant \( \sigma \) belongs to \([0, 1]\) interval, and \( F(k_{\text{opt}}^{\text{opt}}) \) is the \( F \) function value corresponding to the best solution \( k_{\text{opt}}^{\text{opt}} \) that is obtained in a current iteration of the optimization process.
d) Main steps in implementing ACO algorithm:

Step 1 – Setting the parameters: the following parameters of the algorithm are set: number of ants that form the colony \( (N_A) \), maximum permissible number of iterations \( (N_{max}) \), values of \( \alpha \) and \( \beta \) parameters, value of the evaporation rate \( \rho \) of pheromones, value of \( \sigma \) constant of pheromones updating for the best solution.

Step 2 – Initiation: the process is initiated by generating an initial random solution \( k^0 \), that is considered also the best possible solution for iteration 1. For this solution we calculate the value of objective function \( F(k^0) \). The amount of pheromones deposited on each \( (j, i) \) element is initialized to \( \tau(j, i) \leftarrow \tau_0 \) value for all the elements of matrix \( M \).

Step 3 – Evaluation of the optimization function: On an iteration \( p \) of the optimization process (where \( p = 1, \ldots, N_{max} \)) ants \( N_A \) are sent out and each of them determines a solution of the problem \( k^{(p,h)} \) (where \( h = 1, \ldots, N_A \)). Values of the objective function are evaluated using relation (1) for all \( N_A \) solutions obtained at each iteration \( p \). The best solution \( k^{opt} \) gathered from all the iterations made, as well as the value of its corresponding objective function \( F(k^{opt}) \) are kept. If the maximum permissible number of iterations is reached, than this will be the optimal solution. If not, the process will continue with the pheromones updating. The amount of pheromones from \( (j, i) \) positions is updated according to step 4. Based on \( \tau(j, i) \) and \( \eta(j, i) \) parameters, probabilities \( p^{(h)}(j, i) \) are calculated for each \( (j, i) \) element, using relation (7). The higher probability \( p^{(h)}(j, i) \), the better chance \( j \) value becomes the solution of component \( k \) from allocation vector \( k \).

Step 4 – Pheromones updating: is made in two stages, at each iteration \( p \), after all \( N_A \) ants had generated a complete solution. The first stage consists of a decrease of the amount of pheromones on each \( (j, i) \) position in accordance with the evaporation rate of pheromones \( \rho \), using relation (9). The second stage involves the pheromones updating for \( (j, i) \) positions corresponding to the best computed \( (k^{opt}) \), using relation (10).

Step 5 – Process ending: the ending condition of the optimization process is given by the maximum stated number of iterations \( (N_{max}) \).

4. CASE STUDY

To exemplify the application of this redundancy optimal allocation model, we consider a turbo generator unit having a boiler of 350 t/h rated mass flow, a 60 MW turbine and a 55 MVA synchronous generator. The characteristics of the auxiliary equipment that serve the unit are presented in Table 1 [22].
Table 1

Characteristics of the auxiliary equipment that serve the turbo generator unit

<table>
<thead>
<tr>
<th>Equipment</th>
<th>( n_i )</th>
<th>( \lambda_i ) [10^-4 h^-1]</th>
<th>( \mu_i ) [10^-4 h^-1]</th>
<th>( C_{ai,j} ) [EUR]</th>
<th>( c_i ) [%]</th>
<th>( d_{ai,j} ) [EUR/h]</th>
<th>( d_{ai,j} ) [EUR/failure]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal mills</td>
<td>6</td>
<td>40</td>
<td>140</td>
<td>350</td>
<td>10</td>
<td>( d_{11} = 96.1 )</td>
<td>( d_{a11} = 2.2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( d_{12} = 144.2 )</td>
<td>( d_{a12} = 2.2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( d_{13} = 288.3 )</td>
<td>( d_{a13} = 2,150.9 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( d_{14} = 288.3 )</td>
<td>( d_{a14} = 2,150.9 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( d_{15} = 288.3 )</td>
<td>( d_{a15} = 2,150.9 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( d_{16} = 288.3 )</td>
<td>( d_{a16} = 2,150.9 )</td>
</tr>
<tr>
<td>Air fans</td>
<td>2</td>
<td>1.2</td>
<td>280</td>
<td>30</td>
<td>5</td>
<td>( d_{11} = 144.2 )</td>
<td>( d_{a11} = 0 ) ( d_{a13} = 2,150.9 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( d_{12} = 288.3 )</td>
<td>( d_{a12} = 2,150.9 )</td>
</tr>
<tr>
<td>Gas fans</td>
<td>2</td>
<td>3</td>
<td>300</td>
<td>30</td>
<td>5</td>
<td>( d_{11} = 144.2 )</td>
<td>( d_{a11} = 0 ) ( d_{a13} = 2,150.9 )</td>
</tr>
<tr>
<td>Feeding pumps for boiler</td>
<td>2</td>
<td>9</td>
<td>100</td>
<td>90</td>
<td>7</td>
<td>( d_{11} = 144.2 )</td>
<td>( d_{a11} = 2.1 ) ( d_{a13} = 2,150.9 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( d_{12} = 288.3 )</td>
<td>( d_{a12} = 2,150.9 )</td>
</tr>
<tr>
<td>Bagger pumps</td>
<td>1</td>
<td>40</td>
<td>120</td>
<td>120</td>
<td>7</td>
<td>( d_{11} = \begin{cases} 144.2 &amp; \text{for } T_e \leq 0.5 \text{ h} \ 288.3 &amp; \text{for } T_e &gt; 0.5 \text{ h} \end{cases} )</td>
<td>( d_{a11} = 0 ) ( d_{a13} = 2,150.9 )</td>
</tr>
<tr>
<td>Pumps for cooling the condenser</td>
<td>2</td>
<td>1.7</td>
<td>55</td>
<td>25</td>
<td>7</td>
<td>( d_{11} = 135.9 )</td>
<td>( d_{a11} = 0 ) ( d_{a13} = 112.5 )</td>
</tr>
<tr>
<td>Pumps for condense</td>
<td>1</td>
<td>2.8</td>
<td>40</td>
<td>25</td>
<td>7</td>
<td>( d_{11} = 249.2 )</td>
<td>( d_{a11} = 112.5 )</td>
</tr>
</tbody>
</table>

Exhaustive Algorithm. In order to certainly identify the optimal solution regarding the allocation of redundancy for the unit auxiliaries, we had developed a calculation program using MathCAD. It travels to all possible states, setting the limit for redundant elements of each level to \( k_i = 9 \) (that means \( 10^7 \) iterations, the running time of the program was approximately 4 hours on a PC having a 1.5GHz processor). The optimal solution that was identified is \( k_{opt} = (3, 1, 1, 2, 3, 1, 2) \), and for which the value of objective function is \( F_{opt} = 8,928,106 \text{ EUR} \). Redundancy allocation for unit auxiliaries is given by \( (n_i, k_{opt}) \): coal mills (6, 3), air fans (2, 1), gas fans (2, 1), feeding pumps for the boiler (2, 2), Bagger pumps (1, 3), pumps for cooling the condenser (2, 1) and pumps for condense (1, 2).

Heuristic Algorithm (HA). By running the calculation program based on the heuristic algorithm presented above (paragraph 3.1) we’ve got the same optimal solution \( k_{opt} \) after 24 iterations (running time on the same PC was less than one second). The evolution of the algorithm towards the optimal solution can be seen in Fig. 2.

Ant Colony Optimization Algorithm (ACO). In order to apply ACO algorithm (paragraph 3.2) the following parameters were set: \( N_{max} = 160, NA = 6, \alpha = 1, \beta = 2, \rho = 0.05, \sigma = 0.075, \tau = 1, \varepsilon = 0.30 \). The optimal resultant solution is the same \( (k_{opt}) \) and was found after 40 iterations (less than 2 seconds). The variation of the objective function \( F(k) \) while running ACO algorithm is also presented in Fig. 2.
5. CONCLUSIONS

We developed a mathematical model (relation 1) to calculate the optimal allocation of redundancy for auxiliary equipment that serves the turbo generator units. Also, we developed a heuristic algorithm (HA) and a metaheuristic one (ACO) to solve the model. These algorithms are relatively easy to implement and they can identify the optimal solution relatively fast (24, respectively 40 iterations) comparing to the exhaustive travel through the possible solutions space, when $10^7$ iterations are necessary.

HA depends on the initial solution, but determines the optimal solution in a very short time. ACO algorithm is successfully applied, having the same short period for computing the optimal solution. Analyzing the manner in which the two algorithms converge towards the optimal solution (Fig. 2), we find out that ACO algorithm reaches nearby the optimum after only few iterations (4 iterations), while HA converges slower towards the optimum, although it reaches quicker.

We believe that the optimal solution we found is realistic, the number of redundant equipment having the same magnitude as the values utilized in real operation. We must point out the major impact that has the acquisition and assembly cost ($C_{aa}$) on the values of optimal solution.

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