OPTIMAL DESIGN OF INDUCTION MAGNETOHYDRODYNAMIC PUMP BY SIMULATED ANNEALING METHOD

KHADIDJA BOUALI1, FATIMA ZOHRA KADID2, NASSIMA BERGOUG3, RACHID ABDESSEMED4

Key words: Magnetohydrodynamics (MHD), Annular induction pump, Global optimization, Simulated annealing method, Objective function, Optimal design, Constrained optimization.

The magnetohydrodynamics (MHD) is an important interdisciplinary field. It is the interaction between an electromagnetic field and an electrically conducting fluid. Electromagnetic pumps are widely used for the transportation of the fluids in a variety of technological processes. The advantage of these devices is that permits the pumping of liquids without moving parts. The design of the pump is considered as an optimization problem where the objective function is the minimum of the MHD pump mass with both geometrical and electromagnetic constraints type. The obtained optimization results using the finite volume method with Matlab software show the performances of the used stochastic simulated annealing method.

1. INTRODUCTION

Magnetohydrodynamics has been applied to a wide spectrum of technological devices, as a cooler for power semiconductors, electromagnetic propulsion.

An MHD converter has its principal scope to transform the mechanical energy, which is stored in the movement of an electrically conductive fluid, in electromagnetic energy. This mechanism makes it possible to directly convert the fluid movement into electricity without passing through turbines as in the case of conventional power stations. It can also be done in the opposite direction, i.e., using of electrical energy to move a conductive fluid in the pump channel [1, 2].

Optimization methods have been applied with great success in electrical engineering design in recent decades. Most of these activities were done using several stochastic methods [3–9].

These methods have a great ability to find the overall optimum of the problem; they do not require initial point, or the knowledge of the gradient of the objective function to reach the optimal solution. However, they require a large number of evaluations before finding the solution of the problem. Amongst the most used stochastic methods, we distinguish simulated annealing developed by Kirkpatrick in 1983.

The physical annealing process is used in metallurgy to improve the quality of a solid. The aim is to achieve a minimum energy state that corresponds to a stable structure of the metal. Starting from a high temperature at which the material is in liquid form, the cooling phase causes the material to regain its solid form by a gradual decrease in temperature.

The simulated annealing method, as all stochastic strategies, is rather simple to implement, stable in convergence, and able to find desired regions with quite good probability.

In the present study, we will optimize an annular induction MHD pump using the stochastic annealing method. Also, the effect of the control parameter (the temperature reduction factor) on the convergence to the global optimum is performed.

2. PRESENTATION OF THE PUMP

The electromagnetic pump for a liquid metal is considered. A schematic view of the pump is shown in Fig. 1. The liquid metal flows along a channel having a cylindrical geometry of annular cross section. A ferromagnetic core is placed on the inner and the outer side of the channel [10].

The principle of the MHD pump (Fig. 1) is similar to that of the asynchronous motor. When power on the inductor it generates a magnetic field $B$ that slides with the synchronism velocity. This fact determines that an electric currents to be induced in the mercury liquid metal. Through the magnetic field generated by this current an electromagnetic force $J \times B$ appears ensuring the flows of the fluid [11, 12].
The properties of the mercury fluid are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mercury solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $\rho$</td>
<td>$13.6 \times 10^3 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>Electrical conductivity $\sigma$</td>
<td>$1.06 \times 10^6 \text{ S/m}$</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>1.55</td>
</tr>
<tr>
<td>Viscosity $\mu$</td>
<td>$0.11 \times 10^{-6} \text{ m/s}$</td>
</tr>
<tr>
<td>Electric current density $J_{ex}$</td>
<td>$4 \times 10^6 \text{ A/m}^2$</td>
</tr>
</tbody>
</table>

3. OPTIMIZATION PROBLEM AND THE SIMULATED ANNEALING METHOD

In the formulation of the optimization problem, it is necessary to define the objective function and the constraints conditions. In this case, we have considered the mass of the MHD pump as the objective function to be optimized whereas geometrical, electrical and electromagnetic conditions are inequalities constraints.

The resolution of the design problem to determine the vector $X$ will be equivalent to the resolution of the optimization problem $(P)$ defined as follows (Fig. 2):

\[
\begin{align*}
\text{objective function} & = \text{Min mass } (X) \\
B(X) & \leq 1.5 \text{ T} \\
J(X) & \leq 6.10^6 \text{ A.m}^{-2}, \\
X_{\text{lower}} & \leq X \leq X_{\text{upper}}, \\
X & = (X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9),
\end{align*}
\]

where

$X_1$: coil’s width,
$X_2$: coil’s length,
$X_3$: channel’s length,
$X_4$: channel’s width,
$X_5$: air-gap’s width,
$X_6$: inductor’s width,
$X_7$: inductor’s length.

The simulated annealing is an empirical (metaheuristic) method inspired by a process used in metallurgy. It is considered a slow cooling process, that allow long enough time for the atoms to redistribute as their mobility decreases. Physically, the natural mechanism of minimizing energy relies on the Boltzmann probability distribution.

This method is transposed into the optimization problem to find the extrema of a function. The method was proposed by Kirkpatrick, it finds its origins in thermodynamics [13].

The analogies between a physical system and simulated annealing are grouped in the Table 2.

<table>
<thead>
<tr>
<th>Physical system</th>
<th>Optimization problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free energy</td>
<td>Objective function</td>
</tr>
<tr>
<td>Coordinates of the particles</td>
<td>Parameters of the problem</td>
</tr>
<tr>
<td>State of low energy</td>
<td>Optimal configuration</td>
</tr>
<tr>
<td>Temperature</td>
<td>Control parameter</td>
</tr>
</tbody>
</table>

In this algorithm, a new configuration is obtained from a small perturbation subjected to the current configuration. This new configuration is accepted with a probability $p = 1$ when the energy difference $\Delta E$ between it and the current configuration is less than zero. In the case where $\Delta E > 0$, the probability of acceptance $p$ is given by an equation based on the Boltzmann law (4).

\[
P = e^{-\frac{\Delta E}{T}},
\]

where $T$ is the temperature (control parameter).

So, accepting an increase in the objective function, will allow the algorithm to come out of a hollow containing a local optimum; what qualifies this method as a global exploration method.

If the temperature is lowered slow enough and well controlled in the simulated annealing method, the objective function will evolve towards a global optimal solution. Otherwise it will evolve to a local minimum if temperature is lowered suddenly (quenching).

The process continues as long as the energy of the system decreases. When the value of the objective function does not change (the energy remains stationary), the process moves to another temperature level (the decrease of $T$ is done according to a impose decay law) until it convergence to the final temperature where the system becomes frozen [14].

The most common law of the variation of temperature is; given by the relation (5) [15, 16]:

\[
T_{k+1} = \lambda T_k,
\]

where $T_k$ is the previous temperature at the step $k$ and $\lambda$ is the reduction factor ($0 < \lambda < 1$).

To change the temperature level, one can simply specify a number of transformations, accepted or not, at the end of which the temperature is lowered.

A high initial temperature is also chosen. This choice is then totally arbitrary and will depend on the decay law used.

As all metaheuristics approaches, the simulated annealing method can be applied in many optimization problems, such as in packet routing in networks, segmentation of images, the problem of the traveling salesman and the problem of the backpack [17, 18].

Figure 3 shows the flowchart of the implemented simulated annealing method.
In Fig. 3 the \( f(X) \) is objective function and \( f_{opt}(X) \) is optimal value of objective function.

4. RESULTS AND DISCUSSIONS

Considering the constraints in a stochastic optimization method are often obtained by using a function of penalties [4], according to which the function to be minimized becomes equal to:

\[
W(X) = f(X) + r \sum_{i=1}^{m} \left[ \max(0, g_i(X)) \right]^2,
\]

where \( f(X) \) objective function without constraints; \( g(X) \) function’s constraints; \( r \) penalty coefficient.

Tables 3 and 4 show the effect of the \( \lambda \) factor reduction on the solution vector and the pump performances.

| Table 3 |
|-----------------|-------|-------|-------|-------|
| \( \lambda \)   | 0.7   | 0.8   | 0.9   | 0.99  |
| \( W(X) \) (kg) | 9.18  | 8.94  | 8.76  | 8.05  |

The initial temperature \( T_0 \) is calculated using the Metropolis criterion [4]. The objective function \( W \) may not reach the value of the global minimum if the reduction factor \( \lambda \) is smaller than one. From Table 3 can be noticed that \( \lambda = 0.99 \) may be a suitable value.

| Table 4 |
|-----------------|-------|-------|
| Parameters      | Before optimization | After optimization |
| \( X_1 \) (m)   | 0.03  | 0.025 |
| \( X_4 \) (m)   | 0.05  | 0.049 |
| \( X_7 \) (m)   | 0.40  | 0.3801 |
| \( X_9 \) (m)   | 0.020 | 0.019 |
| \( X_1 \) (m)   | 0.004 | 0.001 |
| \( X_4 \) (m)   | 0.2   | 0.1901 |
| \( X_7 \) (m)   | 0.37  | 0.365 |
| Iron mass (kg)  | 2.2523 | 2.0621 |
| Coil’s mass (kg)| 0.1487 | 0.1525 |
| Mercury’s mass (kg)| 6.6261 | 5.8354 |
| Pump’s mass (kg)| 9.0271 | 8.05 |

The choice of the initial high temperature \( T_0 = 100 \) has shown a good convergence to the optimal point and a good reproducibility of the results.

The presented results show that the optimal solution is given for values of \( \lambda = 1 \), which offers a comparable precision and reliability.

Using the obtained optimal dimensions vector, we present the 2D numerical modeling of the electromagnetic phenomena using the finite volume method (FVM) in cylindrical coordinates.

Figures 4–7 present respectively, the distribution of the magnetic vector potential, the magnetic induction, the current density and electromagnetic force with and without optimization in the channel of the MHD pump.

The hydrodynamic model of the MHD pump is based on the Navier-Stokes equation. The solution of the flow equations allows the determination of the velocity and the pressure in the channel of the MHD pump.

Figure 8 presents the variation of the velocity with and without optimization in the pump channel. We note that the velocity of the fluid passes through a transitional period and
then stabilizes as in all the electrical machines. The velocity increases as we advance in the channel.

Figure 9 shows the pressure variations with and without optimization in the channel. It is found that the pressure increases as we advance in the channel. Moreover, the shock values become important in case of a shorter transient state.

By comparing the obtained results in Figs. 7–9, it is noticed that the values increase, this is due to the use of the optimization methods.

5. CONCLUSION

This work presents a design optimization procedure for an annular induction MHD pump using simulated annealing method with constraints. This method has several advantages such as: a good quality solution where the constraints can be easily introduced. Also, the temperature parameter is considered as criteria of the optimization study. The obtained results show that the optimized pump has an improved performance. Also can be noticed that the simulated annealing method may be used successfully in optimization problems.

Received on January 28, 2018

REFERENCES


