INVERTER-INDUCTOR CIRCUIT FOR EDDY CURRENT TREATMENT OF FERROMAGNETIC PIECES

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This paper presents an effective procedure for analyzing the operation of the medium frequency heating system for ferromagnetic pieces. Given the non-sinusoidal shape of the output voltage of the inverter, it was taken into account the harmonics of capacities within the system, selecting the most important ones. At the working frequency of the installation, the depth of penetration of the electromagnetic field is very low compared to the other geometric dimensions and therefore it is suggested the adoption of a uniform environment for this superficial area. The thermal diffusion problem has been solved analytically, using the separation of the solution in spatial eigenfunctions.

1. INTRODUCTION

In order to account for the nonlinearity of the $B-H$ relationship and the pronounced non-sinusoidal form of the inverter’s voltage, the speciality literature provides some important methods, used for solving the eddy currents problem. The static permeability method, corrected by several criteria [1], is used by most commercial software (FLUX, COMSOL etc.). The phasor representation of the electromagnetic field can also be used. The harmonic balance method, which can be relatively easily applied to non-linear electric circuits, raises significant problems in the case of electromagnetic field issues [2]. A huge nonlinear algebraic system, involving all Fourier series coefficients must be solved. In the case of the brute force method, which aims to obtain the asymptotic solution to the electromagnetic field problem, it is difficult to formulate the boundary conditions for the considered inductor-inverter circuit. The only method that seems to be particularly effective in treating the periodic nonlinear regime is described in [3] and is based on the polarization method [4].

For medium frequency, the depth of penetration of the electromagnetic field in ferromagnetic pieces is very small compared to other geometrical dimensions of

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the piece. For example, at 30 kHz, the depth of penetration is less than 0.2 mm before the Curie point and less than 1 mm above the Curie point. For numerical methods based on finite element (FEM), this feature appears as a singularity, requiring a highly nonuniform mesh for space discretization, leading to an ill-conditioning matrix system. The only way one can overcome this drawback is the integral equation of eddy currents. In order to have a simple expression of the kernel equation, with the possibility of integration on sub domains, it is necessary to use the polarization method, and the magnetic permeability of the computing media to be the one of the vacuum [3, 5]. In the above mentioned works, the problem of electromagnetic field is not coupled to that of the circuit, the excitation being represented by the coil imposed current.

In this paper, it was adopted a simple procedure for solving the problem of electromagnetic field coupled with the thermal diffusion problem in the heating system powered by a periodic non-sinusoidal voltage inverter. Just the small depth of penetration is exploited for an easier solving of the electromagnetic-thermal coupled problem. High computation speed allows the find optimal constructive solutions for the desired heating system.

2. EDDY CURRENT PROBLEM

The ferromagnetic piece to be heated has a circular section of radius $a = 4$ mm, it is placed in a coil with the inner radius $R_i = 5$ mm, with a number of $N = 4$ turns and the length $L = 40$ mm. The depth of penetration of the electromagnetic field $\delta$ in the conducting half space verifies:

$$1/\delta = \alpha = \sqrt{\omega \mu \sigma / 2},$$

where $\mu$ is the magnetic permeability of the conducting material (assuming that it is linear and homogeneous), $\sigma$ is conductivity and $\omega = 2\pi f$ is pulsation, corresponding to the frequency $f = 30$ kHz. At this frequency, the penetration depth is less than 1 mm, for the iron material, with $\sigma = 10^7$ S / m, at a temperature above the Curie point, where the relative magnetic permeability is the one of the vacuum. Below the Curie point, the depth of penetration is below 0.05 mm, if the relative permeability is over 400. Specific loss through eddy currents is given by relation:

$$p = \sigma E_0^2 e^{-2\alpha z} = \sigma E_0^2 e^{-2\alpha / \delta}$$

where $E_0$ is the tangential component of the electric field at the boundary of the semi space ($z = 0$). As the dimensions of the piece are larger than the depth of penetration (otherwise surface heating can not be achieved), it is assumed that the penetration of the electromagnetic field in the piece is generally done straight from the semi space. In addition, we admit that the area where the electromagnetic field concentrates has constant magnetic permeability. Above the Curie point, the assumption corresponds to reality. Below the Curie point, the approximation is
acceptable because: the diffusion of temperature in the surface region where the electromagnetic field is located takes place immediately (see Chapter 5); as regards the induced currents and the losses, only the small area where the electromagnetic field presents relatively important values is important. Since this work takes into account the circuit which supplies the inductor in pulses, considering the non-linear \( B-H \) characteristic would greatly complicate the solving of the electromagnetic field problem. It should be emphasized the fact that most of the times, the piece reaches the Curie point, but it can not overcome it, therefore it can not reach the temperature required for the heat treatment. Thus major importance should be given right to the area where the ferromagnetic material is linear.

In order to simplify the presentation, in the following is used the geometric measurements related to the radius of the piece section: \( \zeta = z / a \), \( \delta' = \delta / a \), \( R'_i = R_i / a \), \( L' = L / a \). As it is used the results from the semi space, the \( r \) coordinate is replaced by \( z \), and then by \( \zeta \). We have considered \( S'_a = \pi a^2 (R_i^2 - 1) = a^2 S'_a \) as the area of the section with air between the piece and the inductor. In this area, magnetic field strength, oriented along the inductor, is constant:

\[
H_0 = (N I) / (a L') .
\]  

(3)

The electric field strength at the surface of the piece verifies the relation:

\[
E_{0} = H_0 (1 + j) \alpha / \sigma = (1 + j) \alpha (N I) / (\sigma a L') .
\]  

(4)

The electromagnetic induction law is applied on the contour formed by the inner circumference of the inductor and the outer circumference of the piece, and gives:

\[
2 \pi a (E_{0} - R'_i E_i) = - j a^2 S'_a \omega \mu_0 H_0 ,
\]  

(5)

where

\[
E_i = \frac{U}{2 \pi a R'_i N}
\]  

(6)

is the electric field strength on the inner circumference of the inductor, oriented along this circumference. Substituting (3), (4) and (6) in (5), the complex impedance of the inductor results:

\[
Z_s = \pi \left[ (1 + j) / (\alpha \sigma \delta') + j \omega \mu_0 a (R_i^2 - 1) \right] N^2 / L' = R_s + j \omega L_s .
\]  

(7)

3. INVERTER-INDUCTOR CIRCUIT

Reference [6] describes and analyses different types of invertors, a class D, unsymmetrical inverter being chosen and achieved. It can be equated with a voltage source that generates voltage pulses.
The inverter-inductor circuit is presented in Fig. 1. It can be further simplified using the Kapp scheme for the adaptation transformer.

In order to account for the non-sinusoidal shape of the source were analyzed the first 25,000 harmonics, retaining a much smaller number (only 11), in the order of their share in relation to peak voltage (> 3.7%).

Obviously, the percentage of harmonics currents in relation to their fundamental may change, given the complex impedance on the harmonics, but we believe that a sufficient number of harmonics has been preserved (their number can be increased without any complication). Circuit analysis is performed only on these harmonics. The power consumed by the inductor can be calculated immediately, for a given value of the piece's magnetic permeability:

\[
P_{tot} = \sum_{k \in \text{selected harmonics}} R_s I_k^2, \tag{8}
\]

where, for harmonic \(k\), \(R_s\) and \(I_k\) are the resistance of the inductor and the current within the inductor, and can be easily determined an major of the temperature limit at which the piece might reach:

\[
\theta_{lim} = \frac{P_{tot}}{\alpha S_{lat}}, \tag{9}
\]

where \(S_{lat}\) is the lateral surface of the part of the heated piece \(S_{lat} = 2\pi a^2 L'\). In reality, the limit temperature is lower because heat also dissipates through the
upper and the lower sides of the piece. Interestingly, choosing the parameters of circuit elements given in Fig. 1, when $\mu_r = 1$, the temperature limit is $\theta_{\text{lim}} = 83.78428^\circ \text{C}$. If $\mu_r = 4000$, $\theta_{\text{lim}} = 3374.52^\circ \text{C}$. It results that the piece is heated to the Curie point, and then the temperature remains constant. Temperatures above $800^\circ \text{C}$ cannot be reached, these being required for the heat treatment, which has been confirmed by experiment. The problem can be solved by changing the inductance $L_1$. Figures 2, 3 and 4, 5 respectively present the graphs of the limit temperature and of the actual value of the primary current (totalling harmonics) in relation to $L_1$, for $\mu_r = 1$ and respectively $\mu_r = 4,000$. For the most convenient values of $L_1$ inductivity, these are situated within the range $L_1 \in [ 4.5 \cdot 10^{-5}, 4.7 \cdot 10^{-5} \text{H}]$.

It should be emphasized here that small changes of $L_1$ inductivity may cause significant changes of limit temperature and of the current in the primary. For example, for $L_1 = 3 \cdot 10^{-5} \text{H}$, we obtain $\theta_{\text{lim}} = 3.5 \cdot 10^6^\circ \text{C}$ and $I = 1,000$ A at $\mu_r = 1$. Even if before the Curie point the current is relatively small (15 A) for $\mu_r = 4000$, the sharp decrease of permeability drastically reduces the load impedance and the significant increase of current threatens the functioning of the inverter.

![Graph](image1.png)

Fig. 2 – Limit temperature for $\mu_r = 1$.

![Graph](image2.png)

Fig. 3 – Current for $\mu_r = 1$.

![Graph](image3.png)

Fig. 4 – Limit temperature for $\mu_r = 4,000$.

![Graph](image4.png)

Fig. 5 – Current for $\mu_r = 4,000$. 
4. THERMAL DIFFUSION

The heating of the piece results from the specific losses near the surface of the piece and, as in the case of the electromagnetic field, we assume a Cartesian coordinate system, temperature $\theta$ depending only on the $z$ coordinate and on time $t$. Fourier equation is:

$$-\frac{\lambda}{ca^2} \frac{\partial^2 \theta(\zeta,t)}{\partial \zeta^2} + \frac{\partial \theta(\zeta,t)}{\partial t} = \frac{1}{c} p(\zeta,t),$$

(10)

where $\lambda = 80$ W/(m·K) is thermal conductivity, $c = 196,500$ J/(m³·K) is the volume heat capacity and $p$ indicates specific losses (2). At $z = 0$, the boundary condition is

$$\alpha \theta / \partial \zeta = a \theta \quad \text{or} \quad \theta = \beta \partial \theta / \partial \zeta = \beta,$$

(11)

where $\alpha = 20$ W/(m²·K) is thermal convection coefficient, and $b = \lambda / (\alpha a)$. To simplify, it was admitted that outer temperature is null, in other words will be determined the temperature difference in relation to the outer environment. For $z = a$, namely $\zeta = 0$, it is obtained

$$\partial \theta / \partial \zeta = 0.$$

(12)

The main disadvantage of the numerical solution of the equation (10) is given by the highly uneven distribution of the specific losses, comparable to an impulse in the vicinity of the surface. The spatial mesh network must take into account this non-uniformity which adversely influences the discretization step in time. To obtain a convenient accuracy, we have solved analytically the equation (10), decomposing the solution in series of spatial eigenfunctions [7]. The operator $-\partial^2 / \partial \zeta^2$ is positively and symmetrically defined on the set of functions defined on the [0,1] interval, with boundary conditions (11), (12) and, therefore, it defines a set of eigenfunctions $\Psi_k$ and eigenvalues $\chi_k$, thus:

$$1 - \frac{d^2 \Psi_k}{d \zeta^2} = \chi_k \Psi_k.$$

(13)

The eigenfunctions are orthogonal in the Hilbert space of the functions defined on [0,1] with boundary conditions (11), (12), with the scalar product $\langle \Psi_k, \Psi_l \rangle = \int_0^1 \Psi_k \Psi_l d\zeta$. If we impose that their time to be unified when they form an orthonormal basis on $H$ we can write the piece temperature in the form:

$$\theta(\zeta,t) = \sum_k C_k(t) \Psi_k(\zeta).$$

(14)
Introducing (14) in equation (10) and taking into account (13), it results:

$$\sum \left( q_k C_k + \frac{dC_k}{dt} \right) \Psi_k = p/c,$$

(15)

where $q_k = \lambda \chi^2/(ca^2)$. Multiplying the scalar with $\Psi_k$, the differential equation for the time function $C_k(t)$ results:

$$\frac{dC_k}{dt} + q_k C_k = p_k,$$

(16)

where $p_k = \frac{1}{c} \int_0^1 \rho \Psi_k d\zeta$. If at $t=0$, $\theta = 0_0$, we have $C_k(0) = \int_0^1 \rho \Psi_k d\zeta$ and equation (16) has the solution

$$C_k(t) = e^{-\chi_0 t} \int_0^t t \rho d\tau + C_k(0)e^{-\chi_0 t} = \frac{p_k}{q_k}(1 - e^{-\chi_0 t}) + C_k(0)e^{-\chi_0 t},$$

(17)

because the electromagnetic field varies in time much faster than in thermal diffusion, and $p_k$ can be considered constant in time step $[0, \Delta t]$. If the heating of the axis is analyzed following the time steps $t_{\Delta}$, where it is considered that material parameters are constant, then, at the beginning of the time step, the initial value of $C$ is given by the final value at the previous step. For the beginning, $C_k(0)=0$.

*Functions and eigenvalues.* The function and eigenvalues equation (13) has the solution $\Psi = A \sin(\chi \zeta) + B \cos(\chi \zeta)$. From boundary conditions it results that $A b \chi = B$ and $-\cos(\chi) + b \chi \sin(\chi) = 0$. Therefore eigenvalues $\chi^2_k$ are the solutions of the transcendent equation

$$\cotg \chi_k = b \chi_k.$$

(18)

And eigenvectors are $\Psi_k = A_k (\sin(\chi_k \zeta) + b \chi_k \cos(\chi_k \zeta))$. From the condition of the null norm $\int_0^1 \Psi_k^2 d\zeta = 1$ it results that $A_k = \sqrt{2/(1 + b + b^2 \chi^2_k)}$.

In order to solve the eigenvalues equation (18), the same manner as in [8] has been adopted. This equation is written in the form: $\tan(\chi_k - \pi/2) = -b \chi_k$, which has a solution for argument $\chi_k - \pi/2$ belonging to each interval $(k-3/2)\pi, (k-1)\pi)$, $k \in N^*$. We write $w_k = \chi_k - (k-1/2)\pi$, therefore $\chi_k = (k-1/2)\pi + w_k$ and the equation above becomes $\tan w_k = -(k-1/2)\pi + w_k$ which has solutions $w_k \in (-\pi/2, 0)$ for each $k \in N^*$. Also it can be write $f_k(w_k) = w_k + \arctg \arctg \left[ ((k-1/2)\pi + w_k) \right] = 0$. The $f_k$ function is Lipschitzian and uniformly
increasing because $\frac{df_k}{dw} \in (1, r_k + 1)$, where $r_k < b \left[ 1 + \left( b(k - 1)\pi \right) \right], \forall k$. If is chosen $\gamma_k \in \left( 0, 2/(r_k + 1) \right)$ then function $g_k(w) = w - \gamma_k f_k(w)$ is contraction. The optimal value for $\gamma_k$ is $\gamma_k = 2/(r_k + 2)$, thus obtaining the contraction factor $\theta_k = r_k/(r_k + 2)$. For each natural number $k$, the $g_k(w)$ function has a fixed point $w_k$, obtained by Picard-Banach sequence: $w^{(n)}_k = g_k \left( w^{(n-1)}_k \right)$. Then, is obtained the eigenvalues $\chi_k$. If the iteration is stopped from Picard-Banach sequence at iteration $n$, the error related to the exact solution can be evaluated using the relation:

$$\left| z_k^{(n)} - z_k \right| < \frac{\theta_k}{1 - \theta_k} \left| z_k^{(n)} - z_k^{(n-1)} \right| = \gamma_k \left| z_k^{(n)} - z_k^{(n-1)} \right|.$$  

For $k > 3$ mostly 4 iterations are enough in order to obtain an almost null error ($<10^{-20}$). The components of specific losses are, taking into account (2):

$$p_k = \frac{1}{c_0} \int p \Psi_k d\zeta = -\frac{c}{\eta^2 + \chi_k^2} \left[ e^{-\eta \left[ b^2 \chi_k^2 + 1 \right]} \sin(\chi_k) - \chi_k \left[ \eta b + 1 \right] \right], \eta = \frac{2}{\delta}.$$

\section{5. COUPLING THE PROBLEM OF THE THERMAL FIELD WITH THE EDDY CURRENTS PROBLEM}

The most important influence is that of the $B-H$ nonlinear feature. If approximating this dependent with a linear one, given by static permeability (chosen at $B = 1$ T), the chart of that permeability in relation to temperature is shown in Fig. 6. The discretization in the time domain is done by dividing the interval $[0, \text{max}_{t}]$ in time steps and applying the following algorithm:

Fig. 6 – Relative permeability dependence on the temperature.

a) At the beginning, temperature is $T_0$ (at the first step, $T_0 = 0$). We also initialize the average temperature in time $T_{m0} = T_0$.

b) Magnetic permeability results as a function of $T_{m0}$. For each of the selected harmonics, it is determined the complex impedance of the inductor (7), the
current in the inductor, the value of the electric field intensity at the surface of the piece (4) and the specific losses (2).

c) Is determined the field temperatures given by each harmonic (19), (17), (14) at the end of the time step. These fields are added together and an average temperature near the border is obtained, for a quarter of the depth of penetration: $T_1$.

It is determined the average on the time step of this temperature $T_{m_1} = \frac{T_0 + T_1}{2}$.

d) If the difference between $T_{m_1}$ and $T_{m_0}$ is above a maximum imposed value $\Delta T_{\text{max}}$, the time step is reduced and $T_{m_0} = T_{m_1}$, returning to step b). If the new difference is smaller than $\Delta T_{\text{max}}$, but smaller than a minimum value $\Delta T_{\text{min}}$, the time step is increased, $T_{m_0} = T_{m_1}$ and is returned to step b). If the new difference is within $\Delta T_{\text{min}}$ and $\Delta T_{\text{max}}$, then is established $T_0 = T_1$ and return to step a).

Taking into account the rapid variation of permeability in the Curie point area, in the interval $[750, 770]^{\circ}C$, then it is used a more limited time step $\Delta t = 0.01$ s, in order to ensure the stability of the numerical calculation of evolution in time. Fig. 7 presents the chart of temperature near the piece surface (average for a quarter of the penetration depth) in relation to time, with initial parameters admitted in the scheme from Fig. 1. Judgment assumed in Chapter 3 is confirmed: the piece quickly warms up to the Curie point, which it can not overpass.

Fig. 7 – The temperature evolution for $L_1 = 0.129 \cdot 10^{-3}H$.

Fig. 8 – The temperature evolution for $L_1 = 54.6 \cdot 10^{-5}H$.

If it is chosen the value $L_1 = 4.6 \cdot 10^{-5}H$, keeping the other parameters unchanged, it is obtained the in time evolution of the temperature, presented in Fig. 8. The effective current is less than 30 A.

6. CONCLUSIONS

This paper presents an analysis procedure used for the heating of the ferromagnetic pieces through eddy currents, which allows consideration of voltage
harmonics at the inverter output and dependence upon temperature of the static magnetic permeability. It shows that small variations of parameters of the inverter-inductor circuit (in the case presented, the $L_i$ inductance) drastically affect the operation of the device, from the case where the Curie temperature cannot be exceeded up to the case where the inverter output current gains tremendous values, hence the importance of an accurate determination of these parameters. The procedure can be extended to pieces with sizes, different shapes and sections than the circular one, chosen in this presentation, admitting instead of radius $a$, a distance that is 3 to 4 times that of the electromagnetic field penetration depth.

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REFERENCES