PHASE SHIFTER BASED ON A RECURSIVE PHASE LOCKED LOOP OF THE SECOND ORDER

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This work describes a new type of a phase shifter based on the recursive phase locked loop (PLL) of the second order. This PLL is described by recursive equations. It represents a new approach to PLL. Although this PLL can be used for different applications, the main emphasis is given to the phase shifting of pulse rate. The region of parameters for the stable system and the other analyses of PLL are made using the Z transform approach. Computer simulations of PLL enable better insight into the PLL properties and confirm the mathematical analyses too. The realization of PLL is described. The oscilloscope picture of the input and output pulse rates, recorded on the realized PLL is presented.

1. INTRODUCTION

Recursive PLL and FLL (frequency locked loop), ref. [1–3], represent completely a new approach to PLL and FLL in the literature in theoretical sense and in the way of implementation. These recursive PLL and FLL are based on either the measurement and processing of the time differences between the input and output periods or the measurement and processing of the input periods and the previous output periods. In the classical PLL, correction of the output signal is based on the phase difference between the input and output signals. Because of that, they were named PLL. In addition, some other kinds of classical PLL and FLL are described in the literature too. They are based on errors that represent the amplitude difference or the frequency difference between the input and output signals. The PLL based on the amplitude errors require large hardware and their precision is limited by analog/digital (A/D) converters. The FLL based on the frequency errors, make only the frequency correction of the output signal, while the phase of the output signal is a random variable and depends on the initial values of the input and output signals. This kind of correction is simpler than the corrections.

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of PLL output, i.e. the corrections of the phase and frequency simultaneously. Regardless of that, FLL are widely applied in the frequency-based systems.

The recursive PLL and FLL differs from conventional PLL and FLL in component parts, the way of implementation, functioning, description and in the way of their analysis. The frequency and phase, which are used in classical PLL, are changed by the period and time difference. Unlike conventional loops, it is possible to predict every step of the transition state of the recursive PLL and, if necessary, to make additional controlled corrections at the output. This can improve the efficiency and variety of PLL applications. Very important advantage of recursive PLL is the fact that the digital measurement of time is much easier and more accurate than the measurement of the phase, amplitude or frequency.

The recursive PLL will expand the area of application of PLL and improve the quality of some existing solutions.

One new precise phase shifter, based on the recursive model of PLL of the second order, is described in this paper. Ref. [1, 2] are also based on the recursive PLL. Ref. [1] describes the tracking application of PLL and ref. [2] describes the phase shifter of the first order. Ref. [3] describes one similar recursive algorithm, which is used for the realization of the software predictor. Reference [4] is also closely related to the way of realization of PLL described in this article. The articles [5–8] represent the wider base of literature. The ref. [9–12] are used for electronics implementation and as mathematical and theoretical base.

2. MATHEMATICAL DESCRIPTION OF PLL

One general case of the time relation between an input signal $\text{Sin}$ and an output signal $\text{Sop}$ of PLL, is shown in Fig. 1. The periods $T_{I0}, T_{I1}, \ldots, T_{Ik}, T_{I_{k+1}}$, and $T_{O0}, T_{O1}, \ldots, T_{Ok}, T_{O_{k+1}}$, as well as the time differences $\tau_{0}, \tau_{1}, \tau_{2}, \ldots, \tau_{k}, \tau_{k+1}$, occur at discrete times respectively $t_{0}, t_{1}, t_{2}, \ldots, t_{k}, t_{k+1}$. The discrete times $t_{0}, t_{1}, t_{2}, \ldots, t_{k}, t_{k+1}$ are defined by the falling edges of the pulses of $\text{Sop}$ in Fig. 1. The natural recursive relation (1), between the variables, yields from Fig. 1:

$$\tau_{k+1} = \tau_{k} + T_{I_k} - T_{O_k}. \quad (1)$$

Fig. 1 – The real time relation between the input and output variables $T_{I_k}$, $T_{O_k}$ and $\tau_k$. 
The main recursive equation describing PLL is given by the expression (2):

$$T_{O_{k+1}} = T + n\tau_k + m\tau_{k+1},$$

(2)

where $T$ is the time constant and $n$ and $m$ are the parameters of PLL. The physical meaning of $T$, $n$ and $m$ will be explained in the chapter, which describes the realization of PLL. According to eq. (1) and (2), PLL has two output variables that describe the behavior of PLL in terms of $TI$. The output variables are $TO(k+1) = f[TI(k)]$ and $\tau(k+1) = f[TI(k)]$.

To analyze the conditions under which the described system possesses the properties of PLL, let us find the Z transform of eq. (1) and (2) respectively:

$$z\tau(z) - z\tau_0 = \tau(z) + TI(z) - TO(z),$$

(3)

$$zTO(z) - zTO_0 = T \cdot z / (z - 1) + n\tau(z) + [z\tau(z)] - z\tau_0,$$

(4)

where $\tau_0$ and $TO_0$ are the initial values of $\tau_k$ and $TO_k$ respectively. Calculating $\tau(z)$ from eq. (3) and changing it into eq. (4), it can be found out:

$$TO(z) = TI(z) \frac{zm + n}{z^2 + z(m - 1) + n} + T \frac{z}{z^2 + z(m - 1) + n} + \frac{z\tau_0(m + n) + z(z - 1)TO_0}{z^2 + z(m - 1) + n}.$$  

(5)

Changing now $TO(z)$ from eq. (5) to eq. (3), it can be calculated:

$$\tau(z) = TI(z) \frac{z}{z^2 + z(m - 1) + n} - \frac{z \cdot T}{z - 1} \cdot \frac{1}{z^2 + z(m - 1) + n} + \frac{z\tau_0(z + m) - zTO_0}{z^2 + z(m - 1) + n}.$$  

(6)

Two transfer functions describing PLL, can be defined from respectively eq. (5) and eq. (6):

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{zm + n}{z^2 + z(m - 1) + n},$$

(7)

$$H_{\tau}(z) = \frac{\tau(z)}{TI(z)} = \frac{z}{z^2 + z(m - 1) + n}.$$  

(8)

Providing that the step function is applied to the input $TI(k) = TI = const.$, it is necessary to change the Z transform $TI(z) = TI \cdot z / (z - 1)$ into eq. (5) and eq. (6). Changing $TI(z)$ into eq. (5) and using the final value theorem, it is possible to find the final value of the output period $TO_{\infty} = \lim_{k \to \infty} TO(k)$, using $TO(z)$:

$$TO_{\infty} = \lim [TO(k)]_{k \to \infty} = \lim [(z - 1) \cdot TO(z)]_{z \to 1} = TI.$$  

(9)
Since the phase difference between $\sin$ and $sop$ is changed by the time difference $\tau(k)$ in these analyses, it is necessary to discover the behavior of the time difference $\tau(k)$. If $TI(z)$ is changed into eq. (6), using the final value theorem in the same way like for $TO_\infty$, it yields:

$$\tau_\infty = \lim_{k \to \infty} \tau(k) = \lim_{z \to 1} [(z-1) \cdot \tau(z)]_{z \to 1} = (TI - T)/(m + n).$$ (10)

The expressions (8) and (9) are valued only if PLL is a stable system i.e. if the poles $|z_1| < 1$ and $|z_2| < 1$, where $z_1$ and $z_2$ are the poles of the transfer function $H_{TI}(z)$ or $H(z)$. The poles $z_1$ and $z_2$ can be found out from eq.: $z^2 + z(m - 1) + n = 0$, i.e.

$$z_1 = \frac{m-1+\sqrt{(m-1)^2+4n}}{2} \quad \text{and} \quad z_2 = \frac{m-1-\sqrt{(m-1)^2+4n}}{2}.$$ (11)

According to eq. (9) and (10), the output period $TO$ tends to the input period $TI$ for $k \to \infty$ and it does not depend on either the initial conditions or PLL parameters $m$ and $n$. On the other hand, the time difference $\tau(k)$ also does not depend on the initial conditions of the variables. The mentioned properties for $TO_\infty$ and $\tau_\infty$ meet the necessary requirements and the described system possesses the properties of PLL. However, according to eq. (10), the final value of the time difference $\tau_\infty$ depends on constant $T$ and the parameters $m$ and $n$, giving additional possibility to regulate $\tau_\infty$ by $T$, $m$ and $n$. This means, for the stable PLL, that the frequency of the output pulse rate $sop$ will reach the frequency of the input pulse rate $\sin$, but these two pulse rates will be shifted in phase. The phase difference between them can be regulated by $T$, $m$ and $n$. Using the conclusions reached, as well as the math analyses, the general model of PLL can be introduced now in Fig. 2. PLL is described by two system states $TO(k)$ and $\tau(k)$. Note that the time $T$ is entered into PLL as a binary word and represented by $T_b$ in Fig. 2.

After analyses of the conditions $|z_1| < 1$ and $|z_2| < 1$, using eq. (11), it can be found out the region of parameters $n$ and $m$, for which PLL is a stable system, Fig. 3. The region is limited by the mathematical straight lines $n = -m, m = m-2$ and $n = 1$. PLL described in [2] is just the special case of PLL described in this article. For the special case $n = 0$, eq. (2) will become the same one described in [2], i.e. $TO(k) = T + m\tau_{k+1}$. In this case, it can be seen from Fig. 3, PLL is stable for $0 < m < 2$, just like in [2]. According to Fig. 3, for the range of parameters, given by $m + n = 1$, eq. (10) will be simplified, i.e. $\tau_\infty = TI - T$. 

![Fig. 2 – The simplified model of PLL described.](image-url)
Phase shifter using recursive phase locked loop of the order II

This means that it is possible to change the time difference $\tau_\infty$ linearly, directly by changing $T$. If $T = T_I$, $\tau_\infty = 0$, the phase difference is zero, what is exactly the same as in conventional PLL. According to Fig. 3, this kind of application is possible to realize for wide range of $m$ and $n$ ($0 < m < 1.5$ and $-0.5 < n < 1$). This kind of application is also possible to perform by PLL described in Ref. [2], but only for one case, i.e. for $m = 1$. Since PLL of the second order offers the choice of wide range of parameters $m$ and $n$, PLL of the second order is more powerful to meet the wider range of requirements related to the applications in noisy environments, the applications of PLL with high tracking abilities, the applications of PLL with the adapted transient times and the others.

3. REALIZATION OF PLL

The hardware organization of PLL is shown in Fig. 4. PLL consists of two up-down counters, programmable period generator (PPG), generator of control signals and logic circuits. The functioning of PPG is described in ref. [2]. Let us remember that PPG is based on up-down counter and that the output period $T_O$ of signal $S_{op}$ is $T_O = N_B t_c$. The variable $N_B$ is the decimal value of binary code $N_b$ and $t_c$ is the period of clock. The relation between $t_c$ and clock frequency $f_c$ is simply $t_c = 1/f_c$. To explain the scheme in Fig. 4, let us change $n = f_n/f_c$ and $m = f_m/f_c$ into eq. (2). After change, eq. (2) will be modified to eq. (12), which will be used for realization of PLL:

$$f_c T_{O_{k+1}} = f_c T + f_n \tau_k + f_m \tau_{k+1}. \tag{12}$$

It follows from eq. (12) that three clock signals $S_c$, $S_n$ and $S_m$ with the frequencies respectively $f_c$, $f_n$ and $f_m$ are to be used in the realization of PLL shown in Fig. 3. Note that the clock signals $S_c$, $S_n$ and $S_m$ are changed by their frequencies $f_c$, $f_n$ and $f_m$ in Fig. 3, intentionally. All of four members of eq. (12) are ordinary numbers. It follows from eq. (12) that $T_{O_{k+1}}$ is generated by PPG using clock frequency $f_c$, time $T$ should be taken in account like binary word $T_b = T/t_c$, $\tau_k$ is measured using frequency $f_n$ and $\tau_{k+1}$ is measured by the frequency $f_m$. All these conclusions are applied in the realization of PLL, shown in Fig. 4. The realization of counters and PPG is based on IC four-bits binary up-down counter CD 40 193.
The generator of the time differences $\tau_+$, $\tau_-$, and control signals $Pr_1$, $Pr_2$ and $Pr_3$, according to its function in PLL, changes the role of a comparator in a standard classical PLL. It generates all outputs using $Sin$ and $Sop$. A similar kind of generator is described in ref. [2]. The only addition in this article is the generation of the control signal $Pr_3$. The functioning of the generator and PLL is shown in Fig. 5. The picture in Fig. 5 is made on the realized eight-bit PLL for $T = T_I$ and for very small value of $T_I/t_c$. The voltage waveforms in Fig. 5 are taken when PLL was in the stable state. Since the time difference $\tau$ can be positive $\tau_+$ and negative $\tau_-$ as well, it was necessary to generate them on separate lines and to provide either addition or subtraction, just like in Fig. 4. It is visible in Fig. 5 that $TO$ tends to reach $T_I$ and to reduce the time difference to zero, but that is not possible because the ratio $T_I/t_c$ is very small. The small ratio between $T_I$ and $t_c$ is chosen to enable the visible width of $\tau_+$ and $\tau_-$. The sign of “$\tau$” is changing every period. It can also be seen that whenever two pulses of $Sop$ occur during a period $T_I$, it means that $T_I > TO$, $\tau$ is positive and next $TO$ is increased. If two pulses of $Sin$ comes during a period $TO$, it means that $TO > T_I$, $\tau$ is negative and next $TO$ is decreased. The previous explanations represent, at the same time, the complete description of the generator functioning. If $T_I/t_c$ increases, the widths of $\tau_+$ and $\tau_-$ would decrease and tend to zero. For the stable PLL, the content of up-down counter 2 represents the measured $T_I$ and, at the same time, the output period $TO$, since $TO = T_I$. 

![Fig. 4 – The principle scheme of recursive PLL circuit organization.](image)

![Fig. 5 – The relation $T_I/t_c$ is small and the way of functioning of PLL is visible.](image)
Every $\tau$ is measured instantaneously in both counter 1 and counter 2, Fig. 4. Measuring $\tau$, counter 2 finishes the calculation of $T_{O_{k+1}}$. Counter 1 memorizes $\tau$, at the same time, but for the next calculation of period $T_{O_{k+1}}$. Pulses $P_{R_1}$, $P_{R_2}$ and $P_{R_3}$, shown in Fig. 4, provide the functioning of PLL. As soon as the calculation of $T_O$ is finished, pulse $P_{R_1}$ presets the content of counter 2 to PPG. Pulse $P_{R_2}$ presets the content of $T_b$ into counter 1. At last, after $\tau$ is added to $T_b$, pulse $P_{R_3}$ presets the content of counter 1 into counter 2 for the next calculation of $T_{O_{k+1}}$.

4. SIMULATION OF PLL

The simulation of PLL functioning has more important aims. The first one is to discover additional properties of PLL and its possible efficient applications. The second one is to enable better insight into the physical procedure and meaning of the variables described. The third one is to prove the mathematical analyses described. All discrete values in simulations were merged to form continuous curves. Note that all variables were expressed using “time units” at the following diagrams. The “time unit” or abbreviated “t.u.” can be, $\mu$s, ms or any other, but assuming the same time units for $T_I$, $T_O$, $T$ and $\tau$. It was more suitable to use just “t.u.” in the analysis. All simulations were performed using eq. (2) and eq. (1).

The simulation of $T_O(k)$, $\tau(k)$ and $E_r(k) = T_I(k) - T_O(k)$ for the input pulse rate $S_in$ with the period $T_I = 10$ t.u., $T = 5$ t.u., than for $n = -0.05$ and $m = 1.05$ is shown in Fig. 6a. Since $m + n = 1$, $\tau_\infty = T_I - T = 5$ t.u., i.e. the input pulse rate is shifted for $360 \cdot 5/10 = 180$ degrees. To show the real time transient state of shifting procedure, the presentation of $S_in$, $S_op$ and $\tau_k$ for the simulated case is shown in Fig. 6b. Period $T_O(k) \rightarrow T_I$ and $\tau(k) \rightarrow 5$ t.u. All initial conditions are presented in Fig. 6. The simulation results prove the correctness of eq. (9) and eq. (10).

![Simulation Diagram](image)

Fig. 6 – a Transition state of PLL shifting for 180 degrees for $(m+n)=1$; b. Real time presentation of $S_in$, $S_op$ and $\tau_k$ for the simulated case. Shaded $\tau_k$ are negative and un-shaded ones are positive.
According to eq. (10), the phase shifting can be performed changing $T$ for $m+n=1$. Another way is to keep $T$ constant and to change $(m+n)$. The simulation of the complete PLL functioning by changing $(m+n)$ for the constant $T=5$ t.u. and $TI=10$ t.u., is shown in Fig. 7. The simulation of $TO(k)$ for the different values of parameters $n$ and $m$ is shown in Fig. 7a. It can be seen, since all parameters $n$ and $m$ belong to the region for the stable PLL, Fig. 3, all output periods $TO_1(k)$, $TO_2(k)$ and $TO_3(k)$ tend to $TI(k)$, but with different transient times. It takes $TO_1$ only two steps to reach the stable state. This proves the correctness of eq. (9) and the previous mathematical analyses. The simulation of the complete PLL functioning by changing $(m+n)$ for the constant $T=5$ t.u. and $TI=10$ t.u., is shown in Fig. 7a. It can be seen, since all parameters $n$ and $m$ belong to the region for the stable PLL, Fig. 3, all output periods $TO_1(k)$, $TO_2(k)$ and $TO_3(k)$ tend to $TI(k)$, but with different transient times. It takes $TO_1$ only two steps to reach the stable state. This proves the correctness of eq. (9) and the previous mathematical analyses.

The simulation of the output period $TO(k)$ for the different values of parameters $n$ and $m$ is shown in Fig. 7a. The time intervals $\tau_1(k)$, $\tau_2(k)$ and $\tau_3(k)$ correspond to, respectively: $n=0$, $m=1$; $n=0.2$, $m=0.5$ and $n=0.5$, $m=1.3$. According to eq. (10), $\tau_{1\infty}=5$ t.u., $\tau_{2\infty}=7.14$ t.u. and $\tau_{3\infty}=2.77$ t.u. The calculated values of $\tau_{1\infty}$, $\tau_{2\infty}$ and $\tau_{3\infty}$, agree with those simulated ones, shown in Fig. 7b, proving the correctness of eq. (10). At last, the phase shifting $Ph(k) = 2\pi \cdot \tau(k)/TO(k)$ is simulated and shown in Fig. 7c. It can be calculated that

$$Ph_1\infty = 2\pi \cdot \tau_{1\infty}/TO_1\infty = 6.28 \text{ rad} \cdot 5/10 = 3.14 \text{ rad}.$$  
In the same way it can be calculated that $Ph_2\infty = 4.48 \text{ rad}$ and $Ph_3\infty = 1.74 \text{ rad}$. The calculated values of $Ph_1\infty$, $Ph_2\infty$ and $Ph_3\infty$ agree with those simulated ones, shown in Fig. 7c.
5. CONCLUSION

The description and illustrations of the realized PLL of the second order represent a new approach to the design of PLL in both theoretical and practical sense. The work demonstrates the techniques for theoretical analyzes, practical development and software simulations of recursive PLL based on the processing of the time intervals between the input and output signals.

The main part of this work is devoted to the development of the phase shifter based on PLL of the second order. The analysis and simulations showed that this phase shifter is more complex for the realization than one described in [2], but it possesses considerably better performances for the applications. Due to choice of wide range of parameters that satisfy the conditions of the stable PLL, this phase shifter is more powerful to meet the requirements related to the applications in noisy environments, the applications of PLL with high tracking abilities, the applications of PLL with the adapted transient times and the others.

The precision of the phase shifting can be very high, because it depends on relation $\frac{T_{O}}{t_c}$. The higher relation $\frac{T_{O}}{t_c}$ provides the higher resolution of shifting.

It is obvious that, depending on the field of application, one should carefully choose the parameters of PLL, making the trade off between them, in order to adapt PLL to the specific application. The analysis of PLL properties should be continued in order to expand the area of its application.

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REFERENCES