DESIGN OF NONUNIFORM DEAD-ZONE QUANTIZER WITH LOW NUMBER OF QUANTIZATION LEVELS FOR THE LAPLACIAN SOURCE

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This paper proposes a nonuniform dead-zone quantizer for quantization of the speech signal, modeled with the Laplacian probability density function. It is consisted of the inner dead-zone part and the outer nonuniform Lloyd-Max’s part. The numerical results provided in the paper show that, for the fixed value of source entropy, the proposed quantizer provides the gain in the quality of the quantized signal in comparison to other quantizer models with low number of quantization levels. Accordingly, one can believe that the proposed quantizer will find its way toward the practical implementation, especially in speech quantization.

1. INTRODUCTION

The dead-zone quantizer designing is a field where constant research is being done, since the incorporation of the dead-zone in the centre of the speech signal quantizer helps with filtering of the low level environmental noise due to its high probability of occurrence near the quantizer zero value [1, 2]. In this way, bringing down the quantizer output to the zero level improves the speech signal quality. The previous facts explain the introduction of the dead-zone in the proposed quantizer design. At the input of the proposed quantizer, consisted of one dead-zone interval and an even number of the Lloyd-Max’s quantization intervals, speech signal is modeled with the Laplacian probability density function (PDF). The main goal of the nonuniform dead-zone quantizer design proposed in this paper is to obtain as high as possible quality of the speech signal for the fixed value of source entropy, when the number of quantizer levels is low. It has already been shown in [3] that, for a given source entropy, the nonuniform optimal Lloyd-Max’s quantizer with low number of quantization levels obtains higher signal quality comparing to the uniform quantizer. The analysis has also been conducted in [1] for the high number
of quantization levels, where it has been shown that for a given source entropy the hybrid quantizer, consisted of the uniform quantizer and the nonuniform compandor, obtains higher signal quality comparing to the uniform quantizer. The motivation for introducing the nonuniform quantizer in the quantizer design proposed in this paper is found in the fact that the nonuniform quantization increases the quality of the quantized signal for the given number of quantizer levels. The previously mentioned Lloyd-Max’s quantizer is optimal for any number of quantization levels, but because of its iterative algorithm complexity it is recommended for use with low number of quantization levels. This is the reason why in this paper the analysis of the proposed quantizer is done for low number of quantization levels.

The content of this paper is organized as follows. After the brief introductory section, the section devoted to the presentation of the nonuniform dead-zone quantizer design and properties follows. In the third section the attention is paid on the numerical results which allow us to compare the proposed quantizer performances with the performances of the uniform, the nonuniform Lloyd-Max’s and the uniform dead-zone quantizer. The main contributions of this paper are pointed out in the concluding section.

2. NONUNIFORM DEAD-ZONE QUANTIZER DESIGN

In this section the nonuniform dead-zone quantizer design is described. The proposed quantizer with the odd number of quantization levels \( N = 1 + 2L \), is consisted of the inner part \((-\Delta/2, \Delta/2)\), called the dead-zone where the reproduction level is equal to zero, and the outer part \((-\infty, -\Delta/2] \cap [\Delta/2, \infty)\) where \(2L\) reproduction levels \(y_i\) and \(2L\) decision thresholds \(x_i\) are positioned. We assume that the reproduction levels and decision thresholds of the considered quantizer are symmetrical. The expressions which define the reproduction levels and decision thresholds, referring only on the positive part of the quantization characteristic, are given by [4]:

\[
y_i = \frac{\Delta}{2} + \frac{\Delta}{2} \int_{x_{i-1}}^{x_i} x p(x)dx, \quad i = 1, \ldots, L, \quad (1)
\]

\[
x_i = \frac{\Delta}{2} + \frac{\Delta}{2} \int_{x_{i-1}}^{x_i} x p(x)dx + \frac{(y_i + y_{i+1})}{2}, \quad i = 1, \ldots, L - 1. \quad (2)
\]
Fig. 1 – The first quadrant of the nonuniform dead-zone quantizer characteristic.

Fig. 1 represents the distribution of the reproduction levels and decision thresholds only in the first, positive, quadrant of the quantizer characteristic. From Fig. 1 it stands that:

\[ y_0 = 0, \quad x_0 = \frac{\Delta}{2}, \quad x_L \to \infty. \]  

(3)

The total distortion, introduced by the proposed nonuniform dead-zone quantizer with \( N \) quantization levels, can be expressed as the sum of the distortions introduced in the inner, dead-zone part, and in the outer, Lloyd-Max’s part:

\[ D = D_{DZ} + D_{LM} = \frac{\Delta}{2} \int_0^{\Delta} x^2 p(x) \, dx + 2 \sum_{i=1}^{L} \int_{y_i}^{y_{i+1}} (x - y_i)^2 p(x) \, dx. \]  

(4)

We assume that the input speech signal of standard deviation \( \sigma \) is modeled with the Laplacian PDF:

\[ p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{|x|\sqrt{2}}{\sigma}\right), \]  

and, accordingly, that the reproduction level from the last quantizer interval is determined as in [5, 6]:

\[ y_L = x_{L-1} + \frac{1}{\sqrt{2}}. \]  

(6)

As our goal is to obtain as high as possible quality of the quantized signal, the quantizer optimization process is based on the minimization of the distortion.

The quality of the quantized signal is along with distortion usually measured by the signal to quantization noise ratio (SQNR), which is, for the unit signal variance, defined by [4]:

\[ \text{SQNR} = 10 \log_{10} \left( \frac{1}{D} \right). \]  

(7)
The validity of the quantizer model has properly been estimated in [3, 4, 7], where the SQNR has been calculated for the given source entropy \( H \). Therefore, our analysis is focused on discovering the quantizer model which obtains as high as possible SQNR for the specified source entropy expressed with:

\[
H = -\sum_{i=1}^{N} P_i \log_2(P_i),
\]  

(8)

where \( P_i \) is the probability that the quantizer input falls in the \( i \)-th quantization interval:

\[
P_i = \int_{x_{i-1}}^{x_i} p(x) dx.
\]  

(9)

The previously described analysis is conducted also for the uniform, the Lloyd-Max’s and the uniform dead-zone quantizer for the assumed Laplacian source. The results of the mentioned analyses are given in the next section of the paper. The following section with the numerical results is fulfilled with the comparative analysis of all the mentioned quantizer models.

3. NUMERICAL RESULTS

The nonuniform dead-zone quantizer, proposed in this paper, is optimized to give as high as possible quality of the quantized signal. This optimization process is less complex than the entropy constrained optimization, where two conditions have to be fulfilled at the same time, i.e. the optimal quantizer should provide the maximal quality of the quantized signal for the minimal source entropy. The result of the later optimization is one, discrete, point in the SQNR(\( H \)) graph. In other words, only one quantizer solution is so obtained. On the other hand, the simple, numerical optimization conducted in this paper allows us to choose the best quantizer for many different specified entropy values.

Because both, the SQNR and the source entropy, differ between quantizers for the same number of the quantization levels, it is more precise to use the continuous, graphical, representation of the dependence of the SQNR on the source entropy, to determine which quantizer represents better solution when the entropy is fixed. When only the pairs of discrete values of the SQNR and the source entropy are known, the use of a special developed criteria, for the correct comparison of quantizers properties, is preferred, like in [8].
Below, this section provides the numerical results obtained according to the analytical expressions exposed in the previous section. As mentioned above, SQNR is used as the proper measure of the quantized signal quality. To compare the performances of the proposed nonuniform dead-zone quantizer having \( N = 3, 5, 7 \) \((L = 1, 2, 3)\) quantization levels, with the performances of the Lloyd-Max’s and the uniform quantizer \([3, 4]\), in Fig. 2 are given functional dependencies of the SQNR on the source entropy \( H \). The highest SQNR that can be obtained with the proposed quantizer for the specified number of quantization levels \( N \), is represented with the curve maximum, where the slope of the SQNR(\( H \)) characteristic is equal to zero. These maximum points match with the points of the Lloyd-Max’s quantizer for the same number of quantization levels, whereas for the fixed values of the source entropy and fixed number of quantization levels between these points the proposed quantizer gives the highest SQNR. In other words, these curves define the SQNR behavior of the proposed quantizer which can be considered as optimized under the entropy constraint.
Fig. 2 shows that the proposed nonuniform dead-zone quantizer, for any fixed value of the source entropy, gives higher SQNR in comparison with the uniform quantizer and the Lloyd-Max’s quantizer. Specifically, from the Fig. 2 one can observe that the best performance, i.e. the highest SQNR for the specified source entropy, in the entropy range from 0 to 0.75 bit/sample obtains the nonuniform dead-zone quantizer with $N = 3$ levels. Similarly, in the entropy range from 0.75 to 1.5 bit/sample the best solution among the observed solutions represents the proposed quantizer with $N = 5$ levels. Finally, for the source entropies higher than 1.5 bit/sample the highest SQNR provides the nonuniform dead-zone quantizer with $N = 7$ levels. In these entropy regions the slopes of the SQNR($H$) characteristics of the proposed quantizer are constant, so it is logical to compare the quantizers separately in each of these regions (Figs. 2–4).

Figs. 3 and 4 give the graphical representations of the functional dependencies of SQNR on the source entropy for the nonuniform dead-zone and the uniform dead-zone quantizer [2]. For $N = 3$ levels, these quantizers obtain the same SQNR in the whole observed entropy range. The maximal gain in SQNR of the proposed nonuniform dead-zone quantizer in comparison to the uniform dead-zone quantizer for $N = 5$ levels is slightly above 0.25 dB, and for $N = 7$ levels the gain in SQNR is slightly below 0.5 dB.

![Fig. 3 – SQNR versus source entropy $H$ for the proposed nonuniform dead-zone quantizer and the uniform dead-zone quantizer.](image)
Fig. 4 – SQNR versus source entropy $H$ for the proposed nonuniform dead-zone quantizer and the uniform dead-zone quantizer.

Also, the nonuniform dead-zone quantizer with $N=5$ levels obtains 1 dB higher SQNR, for the source entropy of 1.25 bit/sample, in comparison to the uniform dead-zone quantizer with $N=3$ levels. By observing just the Fig. 4 it can be noticed that the proposed quantizer for $N=7$ levels obtains higher quality for 1.5 dB in comparison to the uniform dead-zone quantizer for $N=5$ levels, for the source entropy of 2 bit/sample.

4. CONCLUSIONS

In this paper the nonuniform dead-zone quantizer has been proposed and analyzed. The performances of the proposed quantizer have been compared with the performances of the uniform, the Lloyd-Max’s and the uniform dead-zone quantizer for the case of low number of quantization levels and for the fixed source entropy. The analyses have shown that for the specified source entropy higher quality of the quantized signal can be obtained with the proposed quantizer when compared to the uniform, Lloyd-Max’s and the uniform dead-zone quantizer. Until recently it was thought that the entropy constrained uniform dead-zone quantizer
has the best performances, but in this paper it has been shown that the nonuniform dead-zone quantizer, when the low and fixed number of quantization levels is considered, obtains higher quality of the quantized signal for the specified source entropy. The difference in SQNR values, achieved by the proposed quantizer and by the uniform dead-zone quantizer, ranges from 0.25 dB for $N = 5$ to 0.5 dB for $N = 7$. One can expect that the nonuniform dead-zone quantizer with a higher number of quantization levels, for example $N = 9$, will provide significantly higher quality of the quantized signal in comparison with the quantizer having $N = 7$ levels, where the amount of the complexity increase is very small. The conclusions derived about the performances of the proposed quantizer would be valid for the case where some of the lossless entropy coding techniques is applied on the output of the proposed quantizer, for example Huffman coding technique, because the lossless coding does not affect the obtained quality of the quantized signal where the average bit rate slightly deviates from the source entropy.

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