USING WAVELET TRANSFORM FOR POWER SYSTEMS

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Key words: Discrete wavelet transform, Distorting regimes, Power quality.

Waveforms from a power plant’s generator are analyzed with Discrete Wavelet Transform (DWT). For the steady state case reference data were calculated (DWT characteristics, details and approximations vectors, RMS value and 3-D tables that can be used to determine faster, in an original way, the parameters of a harmonic distortion). An algorithm is proposed for the evaluation of medium-range unsteady state disturbances and for the recalculation of some indices that characterize the monitored signal (e.g. the correct RMS value, parameters of a harmonic distortion etc.).

1. INTRODUCTION

The indicators for power quality can be used for the quantization and evaluation of waveforms from power systems. The Fast Fourier Transform (FFT) can provide accurate results only for steady waveforms that obey certain requirements. At unsteady state waveforms, even under sinusoidal operating conditions, the use of FFT results into significant errors, owing to spectral leakages, in the same time being unable to provide any time-related information (only the magnitude-frequency dependency is determined) [5].

2. ASPECTS OF DISCRETE WAVELET TRANSFORM UTILIZATION

The wavelet transform packet can represent the waveforms in electric systems with the preservation of both time and frequency information.

When the DWT is used, the original waveform \( S \) is decomposed in approximations and details in the first stage. Afterward successive

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decompositions of the approximations are made, with no further decomposition of the details. Thus a Multiresolution Analysis (MRA) is made (Fig. 1a) [2]. The most significant frequencies from the original signal appear with high magnitudes in that specific region of the DWT signal including them, with the preservation of their time localization, unlike the case when FFT is used [8]. The procedure provides a good time-resolution at high frequencies and a good frequency-resolution at low frequencies [9].

A correct decomposition requires a proper selection of the Wavelet family, of the Wavelet mother and of the levels’ number. Therefore one considered the difference between the energies of signals of adjacent DWT tree’s levels. Further on this difference will be called “energy deviation” and will be calculated as:

$$\Delta E_i = E_i / E \times 100$$

where $E$ represents the decomposed signal’s energy (from the upper level) and $E_i$ represents the energy of the coefficients from the adjacent lower level [12]. These energies can be calculated for the continuous and discrete case respectively:

$$E_i = \int_R c_i(t) \, dt$$
$$E_i = \sum_{n \in N} c_i^2(n),$$

where $c_i$ represent the coefficients obtained from the decomposition [12]. The proper Wavelet family and Wavelet mother obey the requirements for minimum energy deviations for all the decomposition levels.

The decomposed signal can be reconstructed from the approximation/detail coefficients using a schema as that from Fig. 1 (b) (‘+’ means “recomposition”).

A complete set of power quality indices was recently conceived [5] for unbalanced three-phase systems in unsteady regimes. For example, Eqs. (3)…(5) can be used to evaluate with DWT the RMS value of a phase voltage, whose instant values $v$ can be expressed as a discrete signal of length $2^N$ ($2^N$ = multiple of the number of discrete values from the signal’s period). The following symbols were used (the examples refer to Fig. 1): $t$ = time; $l$ denotes a “non-zero” node with details ($l = 2$ corresponds to $D_2$); $j$ = the highest level within the tree ($j = 3$); $k$
is used as index in vectors; $a_j^{(0)}$ means “approximation vector for level $j$”, corresponding to “node 0” (in Fig. 1 it is $A_3$); $d_l^{(n)}$ means “detail vector for the level $l$, non-zero node”; $\phi / \psi$ are used to implement the scaling functions. Based on the approximation and detail vectors generated from its DWT decomposition, we have:

$$v(t) = \sum_{k=0}^{2^{N-j-1}} a_j^{(0)}(k) \phi_{j,k}(t) + \sum_{l=1}^{j} \sum_{k=0}^{2^{N-l-1}} d_l^{(n)}(k) \psi_{l,k}(t) = v_j^{(0)}(t) + \sum_{l=1}^{j} v_l^{(n)}(t). \quad (3)$$

The RMS value of $v$ is denoted by $V$ and can be calculated with [5]:

$$V = \sqrt{\frac{1}{2^N} \sum_{k=0}^{2^{N-j-1}} (a_j^{(0)}(k))^2} + \frac{1}{2^N} \sum_{l=1}^{j} \sum_{k=0}^{2^{N-l-1}} (d_l^{(n)}(k))^2 = \sqrt{(v_j^{(0)})^2 + \sum_{l=1}^{j} (v_l^{(n)})^2}. \quad (4)$$

The RMS value of $v$ can be also expressed as the sum between the RMS value corresponding to the “zero-node” vector ($V_{NZ}$) and the RMS value corresponding to the “non-zero-node” vector ($V_{NNZ}$) [5]:

$$(V)^2 = (V_{NZ})^2 + (V_{NNZ})^2 \text{ where } (V_{NZ})^2 = (v_j^{(0)})^2, \ (V_{NNZ})^2 = \sum_{l=1}^{j} (v_l^{(n)})^2. \quad (5)$$

3. ANALYSIS USING DWT FOR STEADY STATE REGIMES

Data corresponding to currents and voltages were acquired with data acquisition systems along an interval corresponding to 16 periods of a sine wave with a frequency of 50 Hz, obtaining sets of 4 000 discrete values (samples). The data (interpolated to get 4 096 points, a power of 2 being imposed by the use of DWT) were used to study the unsteady state distorting regimes from the secondary winding of the excitation system used to supply the main generator from a power plant [6] using DWT and FFT. A detailed DWT analysis of the current through the 3rd phase was made.

The following terminology was used: steady state refers to a quantity derived from a signal with a frequency of 50 Hz, altered systematically by the same “harmonics” (signals with frequencies multiples of 50) and “interharmonics” (signals with frequencies not dividing by 50). The “unsteady” state refers to a steady state altered for a small period by a non-repetitive disturbance.

The data for the steady state case were processed using an original Matlab program in order to determine the proper order of the Daubechey mother function to be used.

Fig. 2 depicts the percent energy deviations for the levels 1, 2 and 7 (as calculated with Eq. (1)). The higher curve corresponds to the global energy (details plus approximations) whilst the other one corresponds only to the approximation coefficients.

The energy deviations are higher for higher orders of Daubechey mother functions and are minimum for the 3rd order, recommending this order as the best candidate for the Daubechey mother function. Similar conclusions are drawn in [10,
The energy deviations for the 7th level (the lower curve) imposed the limit of 7 as the highest decomposition level.

Fig. 2 – Percent energy deviations for the DWT decomposition, levels 1, 2 and 7.

Applying the inverse of DWT to reconstruct the signal using the approximation and detail coefficients, one determined 43 “approximation error” waveforms (one per each possible Daubechy mother’s order within the range 3…45). Every „approximation error” waveform, representing the difference between the original and reconstructed signals, was used to provide the absolute approximation error, with a maximum denoted by $E_{\text{Max}}$, $i \in [3...45]$. All the values of $E_{\text{Max}}$ were found to be lower than $10^{-4}$ A (6 orders of magnitude less than the decomposed signal’s order), significantly larger values being noticed for Daubechy mother’s orders higher than 20. For the 3rd order, the error is $9.03 \times 10^{-10}$.

This small error and the energy deviations for all levels recommend the Daubechy mother of 3rd order as the best candidate for the DWT decomposition. Fig. 3 depicts representative images extracted from the windows displayed by the program to represent the discrete values of the signal and respectively of the
vectors representing the approximation and detail coefficients, for the 1st and 4th decomposition levels. The conventions used for labeling in Figs. 3–8 are: “s.i.” means “sample’s index”, whilst “c.i.” means “vector’s component index”. The approximation and detail vectors for the steady state are stored in a file for further use.

Fig. 4 – Analysed signal, distorted by a random distortion (unsteady state).

Fig. 5 – Detail vectors (steady and unsteady states, first 2 levels), partial representation.

4. ANALYSIS USING DWT FOR UNSTEADY PERIODICAL REGIMES

For unsteady periodical regimes (USPR) our program performs an analysis of an unsteady signal using DWT based on a 3rd order Daubechy mother function. The studied signal (Fig. 4) was obtained substituting a random signal from the steady state signal. Using the reference data generated for the steady state periodical regime (SPR), the program displays simultaneously the approximation and detail vectors for each level, for the unsteady periodical regimes. Fig. 5 depicts the vectors corresponding to the details for the first two levels (stars corresponds to details in USPR. The regions marked by ellipses corresponds to the values where differences appear. Because in USPR the signal is diminished during the 3rd period with random values, its RMS value should be accordingly diminished. Table 1 confirms this assumption. The last row contains the signal’s RMS value calculated with FFT for the SPR [6]. For 4096 calculation points, the RMS value for SPR calculated with DWT is higher with 2.5 % than the RMS value calculated with FFT (relative to the SPR value, calculated with DWT), but the difference is reduced when more calculation points are used.
<table>
<thead>
<tr>
<th>Calculation method</th>
<th>Regime</th>
<th>$I_{ro}$ [A]</th>
<th>$I_{rn}$ [A]</th>
<th>$I_{ef}$ [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWT</td>
<td>Unsteady</td>
<td>8.475</td>
<td>27.195</td>
<td>28.485</td>
</tr>
<tr>
<td></td>
<td>Steady (reference value)</td>
<td>8.441</td>
<td>27.366</td>
<td>28.639</td>
</tr>
<tr>
<td>FFT</td>
<td>Steady</td>
<td>-</td>
<td>-</td>
<td>27.913</td>
</tr>
</tbody>
</table>

5. DETECTION AND ANALYSIS OF STEADY STATES WITH HARMONIC DISTORTIONS

Repeated harmonic distortions induced in a monitored signal are detected as its instantaneous values record deviations from the expected values. Because the monitored signal’s RMS value is also affected, the approximation and detail coefficients must be recalculated as they are required for the recalculation of the RMS value. As the information on the distorting signal must also be provided, should be very useful to get a correlation: modification of detail coefficients versus characteristic features of the harmonic distorting signal (phase difference with respect to the beginning of the period of the monitored signal, magnitude and frequency).

A study was made, concerning the influence of harmonics with magnitudes from the set \{5, 10, 15, 20\}, phase differences varying in steps of $\pi/12$ within the range $[-\pi, \pi]$ and frequencies calculated with the expression $i \times 50$, $i = 3…40$.

For each possible combination of parameters, the distorting harmonic signal was superposed over the monitored signal along the last period from the analyzed interval.

Fig. 6 depicts a signal altered in the last period by a harmonic signal with a magnitude of 5 A, phase difference of $11\pi/12$ and frequency of 200 Hz.

The modified approximation and detail coefficients (and the difference between the detail coefficients for the unsteady state case and the detail coefficients for the steady state case) were calculated for each combination.

One calculated all the differences $\text{diff}_{lh_n} = cD_{lh_n} - cD_{lh_n}$, where $cD_{lh_n}$ represents the detail coefficients for the $l$-th level, the $h$-th harmonic and a phase difference equal to $u \cdot \pi/12$ ("$n$" is used to denote the initial signal).

Fig. 7 depicts the differences for the harmonic orders 3 and 40, 1-st and 4-th levels.

Two aspects were considered when certain features were selected for analysis:

- the shape of the curves obtained as the difference between the detail coefficients follows the rule “to a higher harmonic order corresponds higher levels where the difference between detail coefficient vectors record the largest number of positive values and values approaching the positive peaks (“close-to-positive-peak”);

- for the same harmonic order, the curves’ shapes seem to vary only with the angle of the distorted signal.
Therefore the following quantities were calculated and saved in 3-D tables:

a) The number of positive values for each specific DWT level, harmonic order and phase difference. The number is saved in the table no_pos, considering that no_pos(l,h,u) is equal to the total number of the positive values from diff_{lhu}.

b) The number of “close-to-positive-peak” values from the difference vector diff_{lhu}. The number is saved in the table close_peak, considering that close_peak(l,h,u) is equal to the number of values from diff_{lhu} with the property: \( \text{diff}_{lhu}(i) \geq 0.9 \times \max(\text{diff}_{lhu}) \).

The first crucial observation (anticipated from the graphic representation) for the values in these 3-D tables (generated for angle increments of \( \pi/12 \) within the range \( [\pi, \pi) \)) was that they are not affected by magnitude. This is due to the shapes’ similarity. For example Fig. 8, depicts the values of diff_{lhu} for the 3 levels, for the 30-th harmonic, angle \( \pi/12 \) and distinct magnitudes (10 and 20 respectively).

The other important observation concerns the 3-D tables’ “fingerprint” relative to the corresponding harmonic order. A dedicated routine was used to verify the assumption that the sequence formed following the pattern \([\text{no\_pos}(\ast,h,u) \text{ close\_peak}(\ast,h,u)]\) for a specific harmonic \( h \) and a specific angle \((u-1) \times \pi/12\) is specific to a single harmonic order, allowing the fast determination of the harmonic order and of the ranges of phase differences for which the pattern is valid.
Fig. 9 depicts (re-arranged for a better understanding) the content of \textit{no\_pos}(\ast, 3, \ast) and \textit{close\_peak}(\ast, 3, \ast), whilst Fig. 10 depicts the content of \textit{no\_pos}(\ast, 40, \ast) and \textit{close\_peak}(\ast, 40, \ast).

![Wavelet transform for power systems](image)

Fig. 8 – Values of \textit{dif}_{\text{flu}} for levels no. 1, 3 and 5, for the 30-th harmonic, angle 11\pi/12, distinct magnitudes (10A and 20A – with higher values), only the non-zero values.

<table>
<thead>
<tr>
<th>Phase difference</th>
<th>\textit{no_pos}</th>
<th>\textit{close_peak}</th>
</tr>
</thead>
<tbody>
<tr>
<td>-\pi</td>
<td>67 34 18 11 7 3 3</td>
<td>1 1 1 4 3 1 1</td>
</tr>
<tr>
<td>-\pi + \pi/12</td>
<td>67 34 18 11 7 3 3</td>
<td>1 1 3 2 2 1 1</td>
</tr>
<tr>
<td>-\pi + \pi/12</td>
<td>67 34 18 10 6 3 3</td>
<td>1 1 1 2 2 1 1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>\pi - 8\pi/12</td>
<td>67 36 19 11 7 7 5</td>
<td>1 1 1 3 1 2 1</td>
</tr>
<tr>
<td>\pi - 7\pi/12</td>
<td>67 34 19 11 8 6 5</td>
<td>1 1 1 2 4 1 1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>\pi - \pi/12</td>
<td>66 34 18 10 7 3 3</td>
<td>1 1 1 2 1 1 1</td>
</tr>
</tbody>
</table>

(The last line (bordered) represents the fingerprint for the 3-rd harmonic, phase difference of \pi – \pi/12)

Fig. 9 – Left – \textit{no\_pos}(\ast, 3, \ast); Right – \textit{close\_peak}(\ast, 3, \ast).

<table>
<thead>
<tr>
<th>Phase difference</th>
<th>\textit{no_pos}</th>
<th>\textit{close_peak}</th>
</tr>
</thead>
<tbody>
<tr>
<td>-\pi</td>
<td>66 36 19 19 7 6 6</td>
<td>16 1 1 10 1 1 1</td>
</tr>
<tr>
<td>-\pi + \pi/12</td>
<td>66 35 19 17 3 5 6</td>
<td>16 10 1 1 1 1 1</td>
</tr>
<tr>
<td>-\pi + \pi/12</td>
<td>67 35 20 16 6 5 6</td>
<td>32 1 1 1 1 1 1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>\pi - \pi/12</td>
<td>66 35 17 19 7 6 3</td>
<td>31 14 1 1 1 1 1</td>
</tr>
</tbody>
</table>

Fig. 10 – Left – \textit{no\_pos}(\ast, 40, \ast); Right – \textit{close\_peak}(\ast, 40, \ast).

Considering the above, the following diagnosis procedure can be followed when a harmonic distortion occurs:

– detect the nature of distortion (easily to reveal by the level of energy deviations as described by Resende, J.W. \textit{et al.} in [13], or by other methods [4, 14]);
– calculate the modified decomposition coefficients and modified RMS value;
– if the distorting signal is revealed to be harmonic, evaluate very fast its “harmonic fingerprint” and compare it to the “reference fingerprints”. Each reference fingerprint is valid for a specific harmonic and range of phase differences (determined in the preliminary stage) and allows the almost instantaneous
determination of the harmonic order and of the phase difference (the last one with an accuracy depending on the range of angles in which the pattern is valid).

The authors made tests on 10 sets, each consisting of 300 harmonic disturbances with random parameters. Patterns generated with a step of $\pi/750$ and a hash function of 5 variables were used. A hit rate of 96.5% (number of predetermined patterns matched) was achieved and the mean execution time for the determination of harmonic distortion was reduced by a factor of 8.37.

6. SCHEMATIC FOR THE CORRECT DETERMINATION OF RMS VALUE AND EVALUATION OF UNSTEADY DISTURBANCES

Fig. 11 depicts a schema that can be used to evaluate the RMS value and medium distortions in unsteady regimes, being especially useful for cases like that from Fig. 4, with unsymmetrical, short term disturbances or for signals with significant interharmonics [7], where it is more difficult to get accurate results using FFT [1].

![Diagram](image_url)
Other power quality related indices can be calculated through further developments of the programs [3]. Direct applications of the programs and of the schema from Fig. 11 are related to the electric quantities monitoring, diagnosis and consumers taxing.

7. CONCLUSIONS

The behaviour in steady state regime of a periodic waveform can provide reference data for the determination, using DWT, of some of its features that enable the evaluation of medium-range unsteady disturbances and the recalculation of some characteristic indices (e.g. RMS values, parameters of a harmonic distortion etc).

In steady state regimes, different RMS values are yielded by different methods of calculation (DWT or FFT). In unsteady state regimes with random disturbances, DWT can be used to determine the RMS value. The correct RMS value is needed to determine with accuracy other indices for the power quality characterization.

The parameters of a harmonic signal that distorts periodically a waveform can be faster and easily deduced from the harmonic’s “fingerprints” calculated with DWT.

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