PARAMETERS ESTIMATION OF AN INDUCTION MOTOR USING OPTIMIZATION ALGORITHMS

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This paper deals with the parameters estimation of a three-phase squirrel cage induction motor integrated in an electric drive system where minimum experimental data are available. For this purpose an experimental test of direct-on-line motor starting under load is carried out and the time variations of phase voltage, current and speed are recorded. Using these values along with the stator phase resistance measured separately in dc, a numerical treatment procedure of the experimental data was developed and other missing parameters of the machine were determined (e.g. magnetization inductance, rotor circuit resistance referred to stator, stator and rotor referred to stator leakage inductances, inertia moment and friction factor). The numerical procedure includes analytical calculations and an inverse problem solving programme that uses optimization algorithms developed under Matlab-Simulink environment.

1. INTRODUCTION

The evolution of modern electric drives is marked by an increase in complexity, reliability and precision of control systems [1, 2] as well as by the development of complex sensorless topologies [3] and embedded systems characterized by high safety, superior fault tolerance and downsizing [4, 5]. The optimization and tuning of complex electric drive systems can be done efficiently by using advanced mathematical models and numerical simulation tools prior to experimental implementation. The numerical simulations offer the advantage of a rapid, cheap and deep analysis of the electric drives systems, providing useful numerical results for the design engineers. However the results accuracy depends strongly on the good estimation or proper measurement of the parameters and performance characteristics of the electric machine to be controlled.

Among the electrical machines a special place is occupied by the induction machine which is the most used in various electric drive systems due to its obvious advantages: high reliability, good efficiency, small cost, ruggedness etc. [6, 7].

The determination of specific parameters of mathematical model of three-phase induction motor is carried out usually by classical no-load and locked-rotor experimental tests. If the induction machine is integrated in a drive system, these tests are in many cases difficult or even impossible to be done without disassembling the electromechanical chain. In this context the elaboration of non-invasive procedures for the determination and estimation of missing machine parameters is extremely useful, even crucial in certain practical applications.

This paper deals with the estimation of the missing parameters of the mathematical model of the three-phase induction machine when it is integrated in a drive system with no possibility to perform standard no-load and locked-rotor tests. To determine the set of necessary machine parameters a direct-on-line motor starting test is done under load and the stator phase voltage, phase current and rotor speed are recorded in transient regime; the stator resistance in dc is measured as well. By processing the recorded data and by solving an inverse problem using an optimization algorithm that control the mathematical model of the machine, the missing parameters (rotor circuit resistance referred to stator, stator and rotor referred to stator leakage inductances, inertia moment and viscous factor) are determined using a dedicated programme developed in Matlab-Simulink.

2. MATHEMATICAL MODEL OF INDUCTION MOTOR

The mathematical model of the induction machine used in the paper is the orthogonal (d-q) model in rotor referential. This model supposes the replacement of the stator three-phase symmetric winding with two single-phase windings oriented along d and q axes; the same equivalence is applied to the rotor circuit as well.

Below are presented the equations of the mathematical model of the induction machine operating as motor and the expressions linking the magnetic fluxes and the currents, by decoupling the paths of the main magnetic field and leakage magnetic field and by neglecting the mutual leakage inductivities, rotor/stator slotting effect and magnetic saturation [6]:

\[
\begin{align*}
V_d &= R_1 i_d + \frac{d}{dt} \Psi_{d1} - \sigma \mu \omega \cdot \Psi_{q1}, \\
V_q &= R_1 i_q + \frac{d}{dt} \Psi_{q1} - \sigma \mu \omega \cdot \Psi_{d1}, \\
0 &= R_2 i_d + \frac{d}{dt} \Psi_{d2}, \\
0 &= R_2 i_q + \frac{d}{dt} \Psi_{q2}, \\
T_e &= \frac{3}{2} \rho \cdot \Psi_{d1} \cdot i_q - \Psi_{q1} \cdot i_d, \\
T_L &= T_e - F \cdot \omega = J \frac{d\omega}{dt}.
\end{align*}
\]

where \( V_d, V_q, i_d, i_q, \Psi_{d1}, \Psi_{q1}, \Psi_{d2}, \Psi_{q2} \) are the voltages, currents and magnetic fluxes along d and q axes corresponding to the stator circuits, \( i_{d1}, i_{q1}, \Psi_{d1}, \Psi_{q1} \) are the values of the rotor currents and magnetic fluxes referred to stator along d and q axes, \( R_1, R_2 \) are the resistances of the stator phase circuit and rotor referred to stator phase circuit, \( \omega \) is the angular rotor speed, \( T_e \) is the electromagnetic torque, \( T_L \) is the load torque, \( J \) is the inertia...
moment, $F$ is the viscous factor, $L_{o1}$, $L_{o2}$ are the leakage inductances of stator circuit and of rotor circuit referred to stator and $L_{m}$ is the magnetization inductance.

Based on the orthogonal $(d-q)$ model of the induction motor expressed in rotor referential, the stator and rotor referred to stator currents ($i_s$ and $i_r$) and all the other quantities (e.g. magnetic fluxes) can be easily computed by Park's transformation [6].

3. DETERMINATION AND ESTIMATION OF MISSING PARAMETERS OF INDUCTION MOTOR INTEGRATED IN AN ELECTRIC DRIVE SYSTEM

The parameters of induction motor integrated in an electric drive system are difficult or even impossible to determine by classical no-load and locked rotor tests since these classical tests would require the dismantlement of the electromechanical chain. That is why in this study a non-invasive method able to estimate the lacking parameters of the mathematical model of the induction machine is proposed based exclusively on the dc measurement of stator phase resistance and on data acquisition of voltage and start-up current and speed.

3.1. DETERMINATION OF INDUCTION MOTOR PARAMETERS

If we consider the first two cycles of the phase current of induction motor during its direct-on-line starting under load we can notice that the motor speed has a small increase and the current has an important amplitude. Thus, we can approximate this time interval by a steady state and in the equivalent electric diagram of the induction motor per phase we can neglect the magnetizing current. If we record the phase voltage ($V_{ph}$), the phase current ($i_{ph}$) and the motor speed ($n$) we can determine the resistance of the rotor circuit referred to stator $R_{2}^*$ and the stator and rotor referred to stator leakage inductances $L_{o1}$, $L_{o2}$ by the expressions:

$$ R_{2}^* = \frac{n_1 - n_{av}}{n_1} \left( \frac{P}{I_f^2} - R_1 \right) $$

$$ L_{o1} \equiv L_{o2}^* = \frac{1}{4\pi f} \sqrt{\frac{V_f}{I_f^2}} - \left( \frac{P}{I_f^2} \right), $$

where $n_1$ is the synchronous speed, $f$ is the supply frequency, $V_f$ and $I_f$ are the rms values of the phase voltage and current; the stator winding phase resistance $R_1$ can be easily measured in dc and the $n_{av}$ and $P$ quantities represent the average motor speed and the active power, which are determined by the expressions:

$$ n_{av} = \frac{1}{T} \int_0^T n \, dt, \quad P = \frac{1}{T} \int_0^T v \cdot i_f \, dt, $$

where $T$ represents the duration of two electric cycles (i.e. 0.04 s for $f = 50$ Hz).

3.2. ESTIMATION OF THREE MISSING PARAMETERS OF INDUCTION MOTOR INTEGRATED IN A DRIVE SYSTEM

In a first case study we suppose there are three missing parameters (magnetization inductance $L_{m}$, moment of inertia $J$ and friction factor $F$) of the induction motor integrated into an electric drive system their evaluation being carried out by solving an inverse problem using a modified Nelder-Mead method (MNMM) implemented in Matlab-Simulink [8–10]. The numerical model of the induction motor that is controlled by MNMM is based on (1), (2) where three parameters are lacking. The inverse problem can be formulated as follows:

$$ F_{obj}(L_{m}, J, F) = \sum_{k=1}^{p} \left( \frac{(i_{smk} - i_{sk})^2}{\max(i_{smk}^2)} + \frac{(n_{mk} - n_{k})^2}{\max(n_{mk}^2)} \right) $$

where $F_{obj}(L_{m}, J, F)$ is an objective function that should be minimized by applying MNMM, $i_{sm}$ and $i_{sk}$ are the measured and simulated phase stator currents respectively, $n_m$ and $n_k$ are the measured and simulated rotor speed respectively, $i_{smk}$ and $i_{sk}$ are the measured and simulated stator currents respectively at time instant $k$, $n_{mk}$ and $n_{k}$ are the measured and simulated rotor speed respectively at time instant $k$, $\max(i_{sm})$ and $\max(n_m)$ are the maximum values of the measured stator phase current and rotor speed respectively, during direct-on-line motor starting.

The objective function $F_{obj}$ is represented by a sum whose elementary term has two components, the first one being related to the measured and simulated phase stator current (i.e. $i_{sm}$ and $i_{sk}$) and the second to the measured and simulated rotor speed (i.e. $n_m$ and $n_k$). The task of MNMM is to find the set of lacking parameters $L_{m}$, $J$ and $F$ that minimize the objective function $F_{obj}$. The optimum point obtained by minimizing the sum of the two terms of $F_{obj}$ means to find the combination of lacking parameters that lead to simulation results of the stator phase current and rotor speed as close as possible to the experimental ones. The ideal case is to find the set of unknown parameters for which the objective function $F_{obj}$ is zero, which means a perfect match between the measurement and the simulation results. This ideal case is a theoretical one because in reality there are relative differences between simulation and experimental results.

The minimization of the objective function $F_{obj}$ is based on Nelder-Mead method (NMM) that is one of the most used optimization algorithm in practical applications. NMM is a deterministic optimization algorithm characterized by high reliability, derivative free, rapid convergence and easy implementation [8, 9]. Other algorithms of deterministic or stochastic types are also available [11, 12]. The main drawback of NMM comes from its very deterministic nature and resides in the impossibility to escape "trap" regions with local minimum points making it unable to continue the search of the global optimum point. In order to surmount this drawback two MNMM variants were implemented (i.e. MNMM1 and MNMM2). Both variants are derived from the original form of NMM and suppose to split the search domain in several subdomains and to restart successively the search of the optimum point from initial points situated in each of those subdomains. In the first
variant (MNMM₁) the search of the optimum point is carried out only in the local subdomain Fig. 1a and in the second variant (MNMM₂) the optimum point is looked for in the entire search domain (i.e. the search is not confined to the local subdomain – Fig. 1b.

![Fig. 1 – Search strategy in case of MNMM; a) MNMM₁ variant; b) MNMM₂ variant.](image)

The search domain (i.e. the limits of variation for each of the three lacking parameters) was defined in our study as follows: \( L_\mu \in [0.098, 0.196] \) H, \( J \in [0.01, 0.1] \) kg.m², \( F \in [0.00319, 0.0319] \) Nms/rad. The computation domain along each axis \( (L_\mu, J, F) \) was subdivided into 5 equally spaced subdomains (i.e. a total of 125 subdomains). After applying MNMM (either first or second variant) by restarting the search process from different initial points (randomly chosen around the center of search subdomains), the best solution is selected from the list of the optimum points found by the two variants of the MNMM. Other procedures for parameters identification of induction motor are presented in [13–17].

### 3.3. ESTIMATION OF FOUR MISSING PARAMETERS OF INDUCTION MOTOR INTEGRATED IN A DRIVE SYSTEM

In the second case study we propose to avoid the procedure described by (3) and (4) used to estimate the resistance of the rotor circuit referred to stator \( R_2 \) by considering this parameter as a missing one. In this case there will be four missing parameters \( (L_\mu, J, F, R_2) \) that are to be found by solving an inverse problem formulated as follows:

Find \( L_\mu, J, F, R_2 \) so as to minimize:

\[
F_{obj}(L_\mu, J, F, R_2) = \sum_{k=1}^{m} \left[ \frac{(i_{sk} - i_{sk})^2}{\max(i_{sk})} + \frac{(n_{mk} - i_{sk})^2}{\max(n_{mk})} \right],
\]

where \( F_{obj}(L_\mu, J, F, R_2) \) is the objective function that should be minimized.

The quantities in (6) are similar to those in (5), the MNMM being used to find the set of lacking parameters \( (L_\mu, J, F, R_2) \) that minimize the \( F_{obj} \) function.

### 4. EXPERIMENTAL MEASUREMENTS AND NUMERICAL EVALUATIONS.

#### RESULTS AND COMMENTS

The experimental tests and numerical simulations were carried out for a squirrel cage three-phase induction motor characterized by the following rated data: rated power \( P_n = 3500 \) W, rated voltage \( V_n = 220/380 \) V, rated current \( I_n = 13.8/8 \) A, rated speed \( n_n = 1420 \) rot/min, rated power factor \( \cos \varphi_n = 0.8 \), rated frequency \( f_n = 50 \) Hz. The rotor phase resistance was measured in dc (voltmeter-ammeter method) and we obtained the value \( R_1 = 1.5 \) Ω.

As presented in Section 3.1, the parameters rotor resistance referred to stator and the leakage inductances can be determined by data acquisition and processing of the stator phase voltage and current as well as the rotor speed during the first two electric cycles of the current during the direct-on-line induction motor startup test under load, Fig. 2.

Using (3) and (4) we obtained the following values for the resistance of rotor circuit referred to stator and for the leakage inductances: \( R_2 = 1.248 \) Ω, \( L_{m1} = L_{m2} = 0.0070 \) H.

The numerical and experimental results of the stator phase current and rotor speed during the direct-on-line motor starting obtained by applying MNMM₁ and MNMM₂ and by measurement are presented in Figs. 3 and 4.

![Fig. 2 – Time variation of stator phase voltage, phase current and rotor speed during the direct-on-line induction motor starting under load.](image)
Fig. 3 – Time variation of stator phase current obtained by simulation (black thin line) and by measurement (grey thick line) during the direct-on-line motor starting under load (3 parameters).

Fig. 4 – Time variation of rotor speed obtained by simulation (black thin line), and by measurement (grey thick line) during the direct-on-line motor starting under load (3 parameters).

Fig. 5 – Time variation of stator phase current obtained by simulation (black thin line) and by measurement (grey thick line) during the direct-on-line motor starting under load (4 parameters).

Fig. 6 – Time variation of rotor speed obtained by simulation (black thin line), and by measurement (grey thick line) during the direct-on-line motor starting under load (4 parameters).
Parameters estimation of induction motor

Classical no-load and blocked rotor tests of the studied induction motor were also carried out and the electrical parameter values obtained experimentally are the following: \( R_2 = 1.3480 \, \Omega \), \( L_\mu = 0.1876 \, \text{H} \), \( L_{o1} = L_{o2} = 0.0077 \, \text{H} \).

These results are close enough compared to those obtained numerically in both studied cases (i.e. for three or four missing parameters of the machine).

5. CONCLUSIONS

This paper proposed a non-invasive procedure that can be applied to determine missing parameters of the mathematical model of the induction motor when the classical no-load and locked-rotor tests of the machine are difficult or impossible to be done without dismantling the electromechanical chain (e.g. when the machine is integrated in a complex/compact drive system).

The original contribution of this paper refers to the numerical procedure that includes analytical calculations and an inverse problem solving strategy based on two optimization methods (MNMM\(_1\) and MNMM\(_2\)) derived from the NMM. This procedure was developed in Matlab-Simulink and by applying it to a 3.5 kW induction motor allowed us to estimate three and four missing parameters of the machine respectively.

The agreement between the numerical simulation results and the experimental ones presented in the paper is very good.

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