FAST CONVERGENCE ADAPTIVE MMSE RECEIVER
FOR ASYNCHRONOUS DS-CDMA SYSTEMS

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Key words: DS-CDMA, MMSE adaptive receiver, Least mean square (LMS), Recursive least square (RLS).

This paper reconsiders the Minimum Mean-Squared Error (MMSE) single user adaptive receiver for the asynchronous Direct Sequence – Code Division Multiple Access (DS-CDMA) system. During the training period the classical MMSE receiver adapts the tap weights once every data bit interval. Our aim is to reduce the overhead introduced during the training period. This will be achieved by adapting the tap weights several times during each bit interval, in an iterative manner. Due to this iterative process, the adaptive filter achieves a faster convergence rate. Hence, the training interval or Multiple-Access Interference (MAI) can be significantly reduced. Nevertheless, this performance improvement is computational expensive.

1. INTRODUCTION

Many of the mobile communications systems are employing the CDMA (Code Division Multiple Access) technique, where the users are transmitting simultaneously within the same bandwidth by means of different code sequences. CDMA technique has been found to be attractive because of such characteristics as potential capacity increases over competing multiple access methods, anti-multipath capabilities, soft capacity, narrow-bandwidth anti-jamming, and soft handoff.

In the DS-CDMA (Direct Sequence – CDMA) system, each code sequence is used to spread the user data signal over a larger bandwidth, and to encode the information into a random waveform [1]. In this case a simple multiplication between the data signal and the code sequence waveform is needed, and the resulted signal inherits its spectral characteristics from the spreading sequence. Hence, because of its linear signal processing function this scheme may be a
subject for possible performance improvements by developing new signal processing techniques for the receiver.

In DS-CDMA, the conventional matched filter receiver distinguishes each user’s signal by correlating the received multi-user signal with the corresponding signature waveform. Hence, the data symbol decision for each user is affected by Multiple-Access Interference (MAI) from other users and by channel distortions. The multiple access interference power depends also on the correlation properties of the chosen spreading sequences [1-6]. Hence, the conventional matched filter receiver performances are limited by its original purpose. It was designed to be optimum only for a single user channel where no MAI is present, and to be optimum for a perfect power control, so it suffers from the near-far problem. These limitations have led to the introduction of an optimum multi-user detection approach, based on Maximum Likelihood Sequence Estimation (MLSE) [2]. The computational complexity of such MLSE is known to grow exponentially with the active number of users, a solution that is not feasible.

In this context, many multi-user receivers have been proposed to overcome the inherent limitations of the conventional matched filter receiver. The use of these multi-user receivers has shown to improve system’s performance, and enhance its capacity relative to the conventional matched filter detection. Unfortunately, most of these multi-user detectors require complete system information on all users.

Implementations of adaptive Minimum Mean-Squared Error (MMSE) receivers in DS-CDMA systems have been analyzed in [3] and [4]. The principle of the adaptive MMSE receivers consists of a single user detector that works only with the bit sequence of that user. In this case the detection process is done in a bit by bit manner, and the final decision is taken for a single bit interval from the received signal. The complexity of an adaptive MMSE receiver is slightly higher than that of a conventional receiver, but with superior performances [3-5, 11]. Besides its facile implementation the adaptive MMSE receiver has the advantage that it needs no supplementary information during the detection process, as compared to the conventional matched-filter receiver.

Within the present paper we aim to improve adaptive MMSE receivers that reduce the MAI in a DS-CDMA system with a faster rate or within a smaller training interval. This will be achieved by adapting the tap weights several times during each bit interval. Due to this iterative process, the adaptive filter achieves a faster convergence rate. This performance is paid by an increased computational complexity.

The paper is organized as follows. In section 2 we describe our asynchronous DS-CDMA system model, both the transmitter and adaptive receiver parts of the scheme. The experimental results are presented in section 3. Finally, section 4 concludes this work.
2. SYSTEM MODEL

In this section we present the asynchronous DS-CDMA system model used in our analysis. The adaptive MMSE iterative receiver is introduced for the asynchronous DS-CDMA system.

2.1. TRANSMITTER MODEL

In the transmitter part of the system, each user data symbol is modulated using a unique signature waveform \( a_i(t) \) (with a normalized energy over a data bit interval \( T, \int_0^T \|a_i(t)\|^2 dt = 1 \)) given by [1]:

\[
a_i(t) = \sum_{j=1}^{N} a_i(j)p_c(t - jT_c), \quad i = 1, K,
\]

where the \( a_i(j) \) represents the \( j \)-th chip of the \( i \)-th user’s code sequence and are assumed to be elements of \{-1, +1\}, and \( p_c(t) \) is the chip pulse waveform defined over the interval \([0, T_c]\), with \( T_c \) as the chip duration which is related to the bit duration through the processing gain \( N (T_c = T/N) \). \( K \) denotes the number of users in the system. In the following analysis we consider binary phase shift keying modulation (BPSK) for signal transmission. Then, the \( i \)-th user transmitted signal is given by

\[
s_i(t) = \sqrt{2P_i} b_i(t) a_i(t) \cos(\omega_0 t + \theta_i), \quad i = 1, K,
\]

where \( P_i \) is the \( i \)-th user bit power,

\[
b_i(t) = \sum_{m=1}^{N_b} b_i(m)p(t - mT), \quad b_i(m) \in \{-1, +1\}
\]

is the binary data sequence for \( i \)-th user, \( N_b \) is the number of received data bits, \( \omega_0 = 2\pi f_0 \) and \( \theta_i \) represent the common carrier pulsation and phase, respectively.

2.2. ADAPTIVE MMSE ITERATIVE RECEIVER

A block diagram of the receiver structure is shown in Fig. 1 [3].

After converting the received signal to its baseband form using a down converter, the received signal is given by:
\[ r(t) = \sum_{i=1}^{K} b_i(t - \tau_i) a_i(t - \tau_i) \cos(\theta_i) + n(t) \cos(\omega_0 t) = \]
\[ = \frac{P_i}{2} \sum_{i=1}^{K} b_i(t - \tau_i) a_i(t - \tau_i) \cos(\theta_i) + n(t) \cos(\omega_0 t), \]
\[ \text{where } N_b \text{ is the number of received data bits, and } n(t) \text{ is the two-sided PSD (Power Spectral Density) } N_0/2 \text{ additive white Gaussian noise (AWGN). The asynchronous DS-CDMA system consists of random initial phases of the carrier } 0 \leq \theta_i < 2\pi \text{ and random propagation delays } 0 \leq \tau_i < T \text{ for all the users } i = 1, K. \text{ There is no loss of generality to assume that } \theta_k = 0 \text{ and } \tau_k = 0 \text{ for the desired user } k, \text{ and to consider only } 0 \leq \tau_i < T \text{ and } 0 \leq \theta_i < 2\pi \text{ for any } i \neq k. \] [3].

\[ \begin{align*}
\sum_{i=1}^{K} b_i(t - \tau_i) a_i(t - \tau_i) \cos(\theta_i) + n(t) \cos(\omega_0 t),
\end{align*} \]

As shown in Fig. 1, the adaptive receiver is modelled as a finite impulse response (FIR) filter with filter taps delayed in steps of the chip duration and total time span equal to the transmitted bit duration.

Assuming perfect chip timing at the receiver, the received signal in (4) is passed through a chip-matched filter followed by sampling at the end of each chip interval to give for the \( m \)-th data bit interval:

\[ r_{m, l} = \int_{mT+(l+1)T_c}^{mT+lT_c} r(t) p(t - lT_c) \, dt, \quad l = 0, 1, \ldots, N-1, \]
where \( p(t) \) is the chip pulse shape, which is taken to be a rectangular pulse with amplitude \( 1 / \sqrt{N} \). Using (5) and taking the \( k \)th user as the desired one, the output of the chip matched filter after sampling for the \( m \)th data bit is given by:

\[
\begin{align*}
    r_{m,l} &= \sqrt{P_k} I_{l,m} a_l(l) + \frac{1}{\sqrt{2N}} \sum_{i=1 \atop \neq k}^{K} P_i \cos \theta_i b_i(m) I_{i,k}(m,l) + n(m,l),
\end{align*}
\]

(6)

where

\[
\begin{align*}
    I_{l,m}(m,l) &= \begin{cases} 
        b_l(m-1) [ e_i a_i(N-1-N_i + l) + (T_e - e_i)a_i(N-N_i + l) ], & 0 \leq l \leq N_i - 1 \\
        b_l(m-1)e_i a_i(N-1) + b_l(m)(T_e - e_i)a_i(0), & l = N_i \\
        b_l(m) [ e_i a_i(l-N_i - 1) + (T_e - e_i)a_i(l-N_i) ], & N_i + 1 \leq l \leq N - 1,
    \end{cases}
\end{align*}
\]

(7)

with

\[
\tau_i = N_i T_e + e_i, \quad 0 \leq N_i \leq N - 1, \quad 0 < e_i < T_e.
\]

(8)

Let us consider the following vectors:

\[
    r(m) = \begin{bmatrix}
        r_{m,0}, & r_{m,1}, & \ldots, & r_{m,N-1}
    \end{bmatrix}^T,
\]

(9)

with \( r_{m,l} \) given by (6). The components of the noise \( n(m,l) \) vector in (6) consists of independent zero-mean Gaussian random variables with variance \( N_0 / (2N) \).

In the training mode, the receiver attempts to cancel the MAI and adapts its coefficients using a short training sequence employing an adaptive algorithm. After training is acquired, the receiver switches to the decision-directed mode and continues to adapt and track channel variations [3-5, 11].

During the training mode, the filter tap weights are adjusted iteratively \( G \) times every transmitted bit interval. At each MMSE iteration \( g \in \{1, 2, \ldots, G\} \) (i.e., \( G \) times for every received bit), the receiver forms an error signal proportional to the difference between the filter output and the known reference signal. This error signal is then used to adjust the filter tap weights using the adaptive algorithm. The error obtained during the \( G \)th iteration of the \( m \)-th data bit is used by the algorithm in the 1st iteration of the \( (m + 1) \)-th data bit. This process is repeated for every received bit until convergence is reached.

Now, let \( w_k(m) \) be the \( N \) coefficient vector representing the adaptive filter weights:

\[
    w_k(m) = \begin{bmatrix}
        w_{k,0}(m), & w_{k,1}(m), & \ldots, & w_{k,N-1}(m)
    \end{bmatrix}^T,
\]

(10)

where the symbol \( m \) denotes the discrete time index of the data bit sequence. The discrete output signal \( y_k(m) \) at the \( G \)th iteration is given by:
Using vector notation, (11) can be written as:

\[ y_k^{(g)}(m) = \mathbf{w}_k^{(g)}(m) \mathbf{r}(m). \]  

(12)

The solution for the optimum filter tap weights that achieve the MMSE are defined by the Wiener-Hopf equation as [7, 8]:

\[ \mathbf{w}_{opt} = \mathbf{R}^{-1} \mathbf{P}, \]

(13)

where \( \mathbf{R} \) is the autocorrelation matrix of the received input signal given by:

\[ \mathbf{R} = E[\mathbf{r}(m)\mathbf{r}^T(m)]. \]

(14)

and \( \mathbf{P} \) is the crosscorrelation vector of the input signal with the desired filter output given by:

\[ \mathbf{P} = E[\mathbf{r}(m)d(m)]. \]

(15)

At the end of each iteration, the receiver forms an error signal \( e_k^{(g)}(m), \)

\[ e_k^{(g)}(m) = b_k(m) - y_k^{(g)}(m), \]

(16)

with \( y_k^{(g)}(m) \) given by (12), and a new filter tap weight vector \( w_k^{(g)}(m) \) is estimated according to the adaptive algorithm. When a new data bit is received, the filter tap weights are adapted in the same manner, with the initial tap weights adapted given by:

\[ w_k^{(0)}(m + 1) = w_k^{(G)}(m), \]

(17)

where \( w_k^{(0)}(m + 1) \) and \( w_k^{(G)}(m) \) represent the initial tap weights at the \((m+1)\)-th received bit, and the final tap weights at time index \( m \), respectively.

The adaptive algorithms used for MMSE receivers can be divided into two major categories [7, 8]. The first one contains the algorithms based on mean square error minimization, whose representative member is the Least Mean Square (LMS) algorithm. The second category of algorithms uses an optimisation procedure in the least squares (LS) sense, and its representative is the Recursive Least Squares (RLS) algorithm. The LMS algorithm with its simple implementation suffers from slow convergence, which implies long training overhead with low system throughput. On the other hand, LS algorithms such as the RLS offer faster convergence rate and tracking capability than the LMS algorithm. This
performance improvement of the RLS over the LMS is achieved at the expense of the large computational complexity.

In the case of LMS algorithm, the new filter tap weight vector \( w_k^{(g)}(m) \) is given by:

\[
 w_k^{(g)}(m) = w_k^{(g-1)}(m) + \mu e_k^{(g)}(m)r(m).
\]  

The parameter \( \mu \) in (18) is the filter adaptation step size chosen to optimize both the convergence rate and the mean squared error [7, 8].

For the RLS algorithm we have to introduce first a new parameter, \( Q_k^{(g)}(m) \). This parameter is iterative computed in the algorithm, starting with an initial value:

\[
 Q_k^{(0)}(m) = \alpha \cdot I_N,
\]

where \( \alpha \) is a positive constant (regularization parameter) and \( I \) is an \( N \)-by-\( N \) identity matrix. A detailed study about choosing the value of \( \alpha \) can be found in [9].

The RLS algorithm can be resumed as follows,

\[
 w_k^{(g)}(m) = w_k^{(g-1)}(m) + \frac{Q_k^{(g-1)}(m)r(m)}{\lambda + r^T(m)Q_k^{(g-1)}(m)r(m)} e_k^{(g)}(m),
\]

\[
 Q_k^{(g)}(m) = \frac{1}{\lambda} \left( Q_k^{(g-1)}(m) - \frac{Q_k^{(g-1)}(m)r(m)r^T(m)Q_k^{(g-1)}(m)}{\lambda + r^T(m)Q_k^{(g-1)}(m)r(m)} \right),
\]

where \( \lambda \) is a positive constant, smaller than unit, which is called the weighting parameter (or forgetting factor). It controls the algorithm’s memory [9, 10].

3. EXPERIMENTAL RESULTS

The asynchronous DS-CDMA system using MMSE iterative receiver presented in the previous section was tested using MATLAB programming environment. A binary-phase shift keying transmission in a training mode scenario was considered. The binary spreading sequences are pseudo-random. The system simulation parameters were fixed as following: \( N = 16 \) and \( K = 8 \). The signal-to-noise ratio (SNR) is 15 dB. The adaptive algorithm is iterated \( G = 10 \) times for each data bit. The mean square error (MSE) was estimated by averaging over 100 independent trials using LMS and RLS adaptive algorithms. The results are presented in Fig. 2 and Fig. 3.
Fig. 2 – Convergence of MMSE receivers using LMS algorithm.

Fig. 3 – Convergence of MMSE receivers using RLS algorithm.

Comparing these results it can be concluded that the RLS algorithm outperforms LMS algorithm. Nevertheless, this performance enhancement is paid by an increased computational complexity.
Finally, we investigated the steady-state BER (Bit Error Rate) performance as a function of the energy per bit to noise power spectral density ratio ($E_b/N_0$) of the iterative MMSE receiver considered above. These results are shown in Fig. 4, where we compared the performance of the iterative MMSE receiver using six consecutive iterations with the conventional adaptive receiver, using LMS algorithm. The simulation results were obtained using 2 000 bits training period for each value of $E_b/N_0$, in order to assure that the steady-state is reached. It is very important to note that under the same conditions the BER is improved every new iteration. Nevertheless, one should make a compromise between the computational complexity and the BER performances.

![Fig. 4 – BER performance of MMSE receivers using LMS algorithm.](image)

4. CONCLUSIONS

The MMSE iterative receiver considered above was shown to improve the asynchronous DS-CDMA system performances. Thus, the MSE is decreased every new iteration by reducing MAI. This decrease offers a faster training mode for the receiver. Hence, the designing procedure may consider one of these two enhancements: to shorten the training sequence for maintaining the same MAI in
the system or to strongly reduce the MAI by keeping the same length of the training sequence. A very important result is that the BER is considerably improved every new iteration. However, this performance improvement is computational expensive. Nevertheless, the systems performances are evaluated by means of simulations. An analytical estimation of BER for this MMSE iterative receiver will be considered in perspective.

Received on 16 July, 2006

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