SYNCHRONIZED PHASOR MEASUREMENTS FOR STATE ESTIMATION

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Synchronized phasors measurements is a modern technology with various applications in power system monitoring and control. This paper presents the application of synchronized phasors measurements to produce more accurate state estimator models. The approach consists in solving the optimal phasor measurement unit (PMU) placement problem in a system that originally has only traditional measurements. The optimisation problem was approached using a genetic algorithm-based method and the IEEE 14 bus test system.

1. INTRODUCTION

State estimators (SEs) play a key role in the real-time operation of power systems. Results produced by SEs (voltages, angles, and active and reactive power flows) represent the input for special contingency analysis methods, optimization procedures, real-time contingency analysis for electricity markets and other applications [1]. State estimation (SE) methods use actual measurements from the network in combination with statistical models to generate an estimation of state variables which best fit metered data. The most popular estimation model is based on the Weighted Least Squares (WLS) method [1, 9], and uses measurements for bus voltage, real and reactive power flows or injections. Traditional approaches to the SE optimization problem use dynamic models based on robust filtering schemes and statistical M-estimation [2], network decomposition techniques [3] or special design methods to keep the system observable during loss of lines and/or measurements [4, 10]. Recently, GPS synchronized phasor measurement data were proposed to be used as input data to the traditional SE model [5, 8]. Phasor measurement units (PMUs) are placed at the buses of transmission substations to measure voltage and current phasors. Compared with other devices, PMUs produce more accurate results due to the synchronization procedure that use high-speed synchronized sampling with 1 microsecond accuracy [7].

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If the state estimator uses only phasor measurement data the estimation model becomes linear, and simpler computing procedures can be applied. However, for large power systems, using SEs based only on phasor measurements can prove a very expansive solution. Therefore, a hybrid solution which uses both traditional and synchronized phasor measurements must be considered. This paper approaches the problem of optimal meter placement as an optimization problem which aims to expand an existing traditional metering configuration through the placement of additional PMUs using a Genetic Algorithm (GA) – based approach. The algorithm aims to find the optimal placement of a given set of PMUs at the buses of a power system with a known conventional metering configuration.

2. TRADITIONAL STATE ESTIMATOR

One of the most frequently used methods for power system state estimation is the WLS model [8]. As measurements, this method uses bus power injections, line power flows and bus voltage magnitudes. The mathematical model aims to find the values of the state variables – bus voltage magnitudes and phase angles – that minimize the weighted sum of the squares of the difference between metered and estimated values:

\[
J([x]) = ([z] - h([x]))^T \cdot [W] \cdot ([z] - h([x])),
\]

where: \([z]\) – metered variables; \([x]\) – system state variables; \(h\) – non-linear functional vector used to compute the estimated value of the measurements; \([W]\) is a diagonal matrix, with components \(W_i\) being equal to the weight associated to measurement \(i\). As shown in [8], the best estimation of state variables \([x]\) can be obtained using as weighting matrix \([W]\) the inverse of the error covariance matrix \([R]\). If measurement variables from vector \([z]\) are statistically independent, matrix \([R]\) has a diagonal form and \(R_{jj} = \sigma_j^2\), where \(\sigma_j\) is the standard deviation of errors associated to measurement \(z_j\), and \(j = 1 \ldots M\), where \(M\) is the number of measurements. The minimization process can be achieved by setting to zero all derivatives of \(J\) with respect to variables from vector \([x]\), and using an iterative method. Thus, if \([x]^k\) is the state variable at the beginning of iteration \(k\), the next estimate \([x]^{k+1}\) should be computed using the following equations:

\[
[x]^{k+1} = [x]^k + [\Delta x]^k.
\]

\[
[\Delta x]^k = G^{-1}([x]^k) \cdot H^T ([x]^k) \cdot R^{-1} \cdot ([z] - h([x]^k)).
\]

In equation (2), \(G([x]^k)\) is the “gain matrix”, and \(H([x]^k)\) is the Jacobian matrix. The iterative process terminates when either a maximum number of iterations has been reached, or the correction term \([\Delta x]^k\) falls below a certain value.
3. STATE ESTIMATOR BASED ON SYNCHRONIZED PHASOR MEASUREMENTS

The traditional WLS model uses only voltage magnitude measurements. Using voltage phase angle measurements in the traditional model is not recommended because the lack of measurements synchronization may introduce major errors. A solution to this problem is to use GPS synchronized phasor measurements technology. More than that, a single PMU installed at a network bus (i.e. the busbar of a substation) can measure not only the voltage phasor of this bus, but also the current phasors in the lines connected to that bus (Fig. 1).

Current phasor measurements can be included in the SE model using the rectangular representation

$$I_{ik} = C_{ik} + j \cdot D_{ik},$$

where:

$$C_{ik} = (g_{i0} + g_{ik}) \cdot U_i \cdot \cos \theta_i - (b_{i0} + b_{ik}) \cdot U_i \cdot \sin \theta_i,$$

$$D_{ik} = (g_{i0} + g_{ik}) \cdot U_i \cdot \sin \theta_i + b_{i0} + b_{ik} \cdot U_i \cdot \cos \theta_i,$$

$$-g_{ik} \cdot U_k \cdot \cos \theta_k + b_{ik} \cdot U_k \cdot \sin \theta_k,$$

$$-g_{ik} \cdot U_k \cdot \sin \theta_k - b_{ik} \cdot U_k \cdot \cos \theta_k,$$

(3)

where \(g_{ik}, b_{ik}\) – real and imaginary parts of admittance of branch \(i-k\); \(g_{i0}, b_{i0}\) – real and imaginary parts of shunt admittance for bus \(i\).

The new algorithm is similar to the basic WLS one, except that it contains specific changes to include phasor measurements, where needed. Thus, the measurement vector \([z]\) will contain entries for current and voltage phasors. A current phasor measurement (based on rectangular representation) is introduced as a 2-component item: the real and the imaginary parts of the current phasor (\(C_{ik}\) and \(D_{ik}\) from equation (3)). On the other hand, since vector \([z]\) already contains voltage magnitude values, a voltage phasor measurement (based on polar representation) will add to vector \([z]\) only one component, the angle of the voltage phasor. The
covariance matrix \([R]\) will also contain more diagonal entries associated to the phasor measurements. The structures of matrices \([H]\) and \([G]\) will change too, but the algorithmic procedure described by equation (2) remains the same.

4. PROBLEM FORMULATION

Consider a system with \(N\) buses and \(L\) lines, which has a limited number of conventional measurements: bus voltage magnitudes, bus power injections and line power flows. For this system, the optimal placement of \(M\) PMUs must be determined, in order to obtain better estimation of the state variables. The optimization problem was solved using a genetic algorithm (GA) approach [6]. The GA was implemented using MATLAB functions that allow introducing, modifying and analysing data and also writing them in files for further use. In first stage a population of possible solution of the problem is generated. Chromosomes will pass through an evolutionary process using selection, crossover and mutation operators, until the optimal solution is reached. For every solution described by a chromosome in the current population, the state estimator function is run to compute the fitness value associated to the given chromosome.

The GA was applied from an initial configuration described by a certain number and types of conventional measurements. This configuration contains voltage and/or power injection measurements in different buses and / or power flow measurements on different branches (lines or transformers). In a more general case, that was not considered in this paper, the initial configuration could contain existing phasor measurements too. The approach proposed in this paper considers that the optimal PMU placement problem starts from this initial configuration that contains measurements already provided by the existing SCADA system. For each possible solution, the basic configuration was updated by adding phasor measurements and the SE function was run again. On the other hand, when placing a PMU measurement in a bus all traditional measurements (if any) from that bus are eliminated, since PMU measurements are more accurate than traditional ones.

4.1. CHROMOSOMES DESCRIPTION

A chromosome which describes a solution to this problem is formed by \(M\) blocks, each one associated to a PMU. A block describes the binary code of the bus where the PMU is placed. Hence, the number of bits in a block and the chromosome length are fixed. For instance, for the IEEE14-bus system, used in testing the proposed model, each block uses a 4 bits representation. Thus, a possible solution for a 3 PMUs placement-problem may use buses 4, 13 and 10, and the chromosome describing this solution will have the following structure:

\[
0 1 0 0 1 1 0 1 1 0 1 0
\]
4.2. CONSTRAINTS

The application of the GA procedures must comply with certain constraints: (i) since the chromosome model can generate binary codes greater than $N$, each PMU-block is tested to verify if encoded number is between 1 and $N$; (ii) all blocks in a chromosome must be different since only one PMU can be placed at a bus; (iii) 2 PMU cannot be placed at buses at the edges of the same power line. Since PMU also gives information about current phasors in the lines connected to one bus, there is enough information to determine the voltage phasor at the other bus.

4.3. THE FITNESS FUNCTION

The accuracy of the SE results were assessed using as reference the power flow for the steady-state conditions. The chromosome’s fitness function $FF$, which measures the accuracy of the SE, was computed as the inverse of the cumulative difference between estimated power flows $S_{ij}^{est}$ and the reference power flows $S_{ij}^{ref}$ on branches linking buses $i$ and $j$, for the steady state conditions, divided by the number of lines $L$ in the system:

$$FF = L \left/ \sum \left| \frac{S_{ij}^{est}}{S_{ij}^{ref}} \right| \right. .$$

In the SE function the weight associated to a measurement reflects the confidence awarded to it or the absence of this measurement. The weights are read out from external data files, and all weights corresponding to conventional measurements associated to a bus where a PMU is placed and to the lines connected to that bus are set to zero. At the same time, new weights for bus voltage magnitudes and angles, and also current flows on the lines connected to that bus are included in the weights matrix.

Measurements were simulated by applying a white noise to the reference variables. Traditional measurements uses errors’ standard deviations between 0.004 and 0.020 p.u., while PMUs data use a value of 0.001 p.u. To take into account the non-uniform distribution of measurement errors in all buses of the power system, measurements have been grouped into three classes, based on the assessment of function (4) computed using the steady-state conditions and state estimates based on random values of measurement data. Three measurement classes were used: small (SV), medium (MV) and large (LV) variance (Table 1).

<table>
<thead>
<tr>
<th>No. PMU</th>
<th>LV</th>
<th>MV</th>
<th>SV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.57</td>
<td>7.66</td>
<td>9.33</td>
</tr>
<tr>
<td>2</td>
<td>16.8</td>
<td>16.16</td>
<td>16.22</td>
</tr>
<tr>
<td>3</td>
<td>17.52</td>
<td>17.51</td>
<td>17.52</td>
</tr>
<tr>
<td>4</td>
<td>17.93</td>
<td>18.12</td>
<td>17.95</td>
</tr>
</tbody>
</table>
Simulations were carried out on the IEEE 14-bus test system from Fig. 2. A limited number of traditional measurements that lead the system to the verge of observability were considered, as follows: a voltage measurement in the swing bus 1; power injections measurements in buses 2, 8, 9 and 14; power flow measurements on lines 1-2, 2-3, 4-5, 4-9, 5-6, 6-11, 6-13, 7-9 and 12-13.

As reference measurements the steady-state conditions presented in the previous section are considered. When the state estimator is run for the reference steady-state conditions, the values of function (4) for all three measurement classes are those from the first row in Table 1. These values were compared with values of the fitness function computed for the optimal solutions generated by the GA in different hypotheses concerning the number of PMUs placed in the system.

The analysis has considered that the above traditional measurements were supplemented with PMUs, using a replacement procedure like the one described in section 4. Taking into account the number of buses in the test system, the analysis has considered three cases for the number of PMUs to be placed in the network: 2, 3 and 4 PMUs. The results obtained for these cases are described in the following paragraphs.

**Case 1: 2 PMUs**

To illustrate the complexity of the optimization problem, a 3-D representation of fitness function was used. The surface plots use on the horizontal axes the number of buses where the 2 PMUs are placed, and on the vertical axis the fitness function. For instance, Fig. 3 presents the surface plot for LV, MV and HV accuracy.
Fig. 3 – Surface plot for: a) LV, b) MV and c) SV accuracy classes. Case 1 – 2 PMUs.
Fig. 4 – Chromosomes from the last generation of the GA, for: a) 2 PMUs, b) 3 PMUs and c) 4 PMUs and the MV accuracy class.
classes. As one can see from these figures, a considerable number of local peaks can be detected on the plots. This is a serious reason to use the GA as a solver for the optimization problem under consideration, since an important characteristic of the GA is that it avoids getting caught in those local peaks and determines the optimal solution, in this case the global maximum of the fitness function.

The evolution of the chromosomes during the run of the GA can be depicted by the pairs of buses were the 2 PMUs are placed, as in Fig. 4.a. The chromosome with the optimal solution appears somewhere between the 15th and the 30th generation. Eventually, the most chromosomes in the last generation describe the optimal solution, namely the placement of the 2 PMUs at buses 4 and 6. The fitness function increases from 7.66 p.u. to 16.16 p.u. (Table 1).

**Case 2: 3 PMUs**

For this case, the chromosomes from the last generation are represented in Fig. 4.b, for the case of the MV accuracy class. The optimal solution consists in placing the 3 PMUs at buses: 9, 11 and 12. This corresponds to an increase of the fitness function from 7.66 p.u. to 17.51 p.u. as shown in Table 1.

**Case 3: 4 PMUs**

Similar to the previous case, only one measurement accuracy class was considered to present the chromosomes from the last generation (Fig. 4.c). The optimal solution found places the 4 PMUs at buses 2, 6, 7 and 14. This time the fitness function increases from 7.66 p.u. to 18.12 p.u., as shown in Table 1.

Data from Table 1 reflects a positive effect of using PMUs as data-sources for the SE. As one can see, when the SE uses only traditional measurements (the first row from Table 1), the variance of the measurement errors influence in a great extent the estimator accuracy (function FF varies from simple to double). On the other hand, using PMUs as data-sources for the SE (the last three rows from Table 1), produces the following effects:

- the estimation accuracy improves significantly (the fitness function increases by 2 to 4 times).
- for a given number of PMUs, the estimation accuracy remains approximately unchanged when measurement error variances vary from LV to SV.

Data from Table 1 show that by increasing the number of PMUs to be placed in the system, the estimator accuracy also increase, but with a decreasing rate.

### 6. CONCLUSIONS

The paper presents the possibility of using PMUs to system state estimation. The results show a significant improvement in the estimator accuracy when
traditional measurement units and PMUs are simultaneously used. The optimization problem for PMUs placement was solved using a GA approach. The accuracy of the estimator was assessed using as fitness function for the GA the inverse of the cumulative differences between estimated and real power flows in the system. PMUs presence produces a 4 time increase of the fitness function.

The approach presented in this paper will be implemented as part of a research project (see the acknowledgement section), which aims to develop new methods and techniques for congestion prediction and management based on synchronized phasor measurements. The experimental stage of this project will include also real life network structures, measurements and operating conditions to be tested with the proposed state estimation approach.

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