METHOD OF EXTRACTING THE EQUIVALENT CIRCUIT OF COMPLEMENTARY SPLIT RING RESONATOR LOADED TRANSMISSION LINE RELYING ON NEGATIVE PERMITTIVITY CHARACTERISTIC

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Key words: Metamaterials, Equivalent circuit of complementary split ring resonators (CSRRs), Split ring resonators (SRRs), Microstrip technology.

In this work, a new approach for extracting electrical parameters of the equivalent circuit of complementary split ring resonators (CSRRs) loaded transmission line, based on the frequency band where negative electrical permittivity occurs, is proposed. For this reason, CSRR coupled to microstrip transmission line has been proposed and the behavior of the frequency band where negative permittivity appears is investigated, in order to specify the fundamental frequencies that are needed to calculate the electrical parameters of equivalent circuit. The procedure provides the electrical characteristics of CSRR (inductance, $L$, capacitance, $C$, and intrinsic resonant frequencies). The obtained equivalent circuit model of CSRR loaded transmission line has been simulated and a good agreement is achieved between the results derived from the simulation and the ones form the equivalent circuit model. Further validation of the usage circuit is confirmed by applying different lengths of transmission line (host line); in this case, the frequency interval of negative permittivity band decreases and shifts to lower frequencies due to change in host line capacitance per unit cell. Nevertheless, the results of minimum insertion loss which were determined by the provided equivalent circuit model (which are proposed by the approach) are remaining compatible with the simulation. Ansoft HFSS (High Frequency Structure Simulator) simulator and Matlab are used for structures' design and analyzing the results.

1. INTRODUCTION

Despite reveling interesting properties of media with simultaneously negative permeability and permittivity (hypothesized by Veselago in 1967 [1]), it has been waited until 2000, when the first experimental proof was carried by Smith et al. [2]. The authentic medium suggested in [2] consists of large combination of metal split ring resonators and wires (SRRs) [3], and due to the negative effective permeability ($\mu < 0$) of the structure a birefringent property of SRRs is created [3–5]. SRR behaves as an $LC$ resonant tank due to distributing of capacitance between concentric rings and overall rings inductance, and this $LC$ resonant tank can be excited by external magnetic flux. Subsequently a second particle has been suggested for effective material design, namely, complementary split-ring resonator (CSRR), which is the negative image of SRR and is depicted in Fig. 1 [6, 7]. It has been demonstrated that CSRRs etched in the ground plane of transmission line media supply a negative effective permittivity ($\varepsilon < 0$) to all media construction [8].

Several geometric forms of CSRR have been suggested by many research groups in the recent past [9, 10]. SRRs and CSRRs (completely built-up with shunt metallic wires or series gaps) have been successfully utilized to the design of planar microwave circuit and devices [11], such as design of compact planar filters with great performances and controllable characteristics [12–15], and design of planar sensors for materials characterization in various applications [16–18]. SRRs and CSRRs have often small electrical size, in which their impact on microwave circuit may be considerable in the future. Considering these all, the electrical characteristics of CSRRs are very important to aid the device design.

Analytical models capable to supply the equivalent inductance and capacitance of both SRRs and CSRRs have been suggested [4, 7]. However, these models are useful under several limited conditions, that is, absence of host line (microstrip transmission line). Thus, the equivalent circuit extraction technique suggested in [19, 20] is of benefit since it directly provides the circuit parameters of those circuits forming. The previous technique has been illustrated in above references and it is reported here for concision in comparing the procedure of this article.

The traditional technique determine the parameters of the circuit model (equivalent circuit of unit cell is $T$ line with negative permittivity) through some major conditions which explained by representation of reflection coefficient of a single unit cell, $S_{11}$, on the Smith chart, in order to obtain the frequency that nulls parallel admittance. Another condition represented by Transmission coefficient, $S_{21}$ is needed to obtain the resonance frequency that nulls parallel impedance. Last condition can be derived from the phase of $S_{21}$, where $\phi(f_{c2}) = -90^\circ$ in which the series and shunt impedances are being equal ($Z_s(j\omega_{2c}) = Z_p(j\omega_{2c})$) [19, 20]. Thus, from these conditions, reactive electrical parameters of equivalent circuit can be determined.

Also, a new circuit model for the microstrip line loaded with CSRR is proposed in [21] and analyzed using similar previous procedure. Nevertheless, the techniques applied in previous references are efficient but did not take into consideration the features that are achieved by specific frequency band of effective negative permittivity and permeability.

![Fig. 1 – Topology of the basic CSRR unit cell.](image)
In this work the all scattering parameters of CSRR loaded transmission line are identified by using specific frequency band interval where negative permittivity occurs and are used to create the equivalent circuit.

2. RESPONSE OF NEGATIVE PERMITTIVITY OF CSRR UNIT CELL

The unit cell of the CSRR loaded microstrip lines with CSRRs etched in the ground plane is shown in Fig. 2, where $L$ and $C$ are the per-unit-cell inductance and capacitance of the host microstrip line, $C_c$ and $L_c$ are total capacitance and inductance of CSRR which represent electrical coupling on the ground. The physical properties of the structure in Fig. 2 are characterized by [22]

$$\varepsilon = \frac{Y_p}{j\omega} = \frac{C(1 - \omega^2 L_c C_c)}{1 - \omega^2 L_c (C + C_c)}, \quad (1)$$

$$\varepsilon < 0 \text{ for } \frac{1}{2\pi \sqrt{L_c (C + C_c)}} < f_e < \frac{1}{2\pi \sqrt{L_c C_c}}. \quad (2)$$

From (1) and (2), Re($\varepsilon$) is negative, then in this case, propagating waves are reflected to establish a standing-wave, hence, transmission zero occurs through the resonator. As a result, CSRR loaded transmission line can also be used to form a band rejected filter with high quality resonator. From (2), it is interesting to point out, the region $\varepsilon < 0$ is a frequency band limited by two critical resonance frequencies, lower and upper frequency ($f_s = \frac{1}{2\pi \sqrt{L_c (C + C_c)}}$, $f_0 = \frac{1}{2\pi \sqrt{L_c C_c}}$). The frequency that nulls the parallel admittance is short circuit frequency, $f_s$ (notice that, this frequency is not the transmission zero frequency of the unit cell) and the frequency that nulls the parallel admittance is open circuit frequency, $f_0$ (notice that, this frequency is not the resonance frequency of the tank (CSRR), but the one for the parallel group formed by $L_c$ and $C_c$).

![Fig. 2 – Topology of the basic CSRR unit cell and its equivalent circuit model.](image1)

In order to highlight the negative permittivity frequency band which is given by lower and upper frequencies ($f_s$, $f_0$), the CSRR unit cell shown in Fig. 2 has been designed and simulated as shown in Fig. 3.

The substrate employed is FR4-epoxy (Epson $\varepsilon_r = 4.4$, thickness $= 1.6$ mm). The microstrip line has a width of 3.083 mm, which corresponds to a characteristic impedance of 50 $\Omega$. The resonant frequency of the CSRRs is around 2.4 GHz (Fig. 3 a). The physical dimensions in this case are $d$, $s = 0.76$ mm, $g = 5.32$ mm (Fig. 1). The width of the slit $g$ at each ring is equal 0.16 mm and the metal thickness $t$ is equal 0.035 mm.

![Fig. 3 – Simulation of unit cell; a) magnitudes of s-parameters ($S_{11}$, $S_{21}$); b) permittivity response.](image2)

Figure 3b shows $\text{Im}(\varepsilon)$ of CSRR unit cell, as well as the interval of negative permittivity band (20.2 % fractional bandwidth) which is limited by lower and upper frequencies, $f_s$ and $f_0$, respectively.

After clarification of negative permittivity band and related critical frequencies, it is necessary to explain the fundamental parameters which are used to extract the equivalent circuit.

Figure 4 shows $\text{Re}(\varepsilon)$ and the $\text{ph}(\varepsilon)$, as well as three frequencies ($f_s$, $f_2$, and $f_0$). The two characteristic frequencies $f_s$ and $f_0$ can be specified as: $f_s$ is the lower critical frequency that nulls parallel impedance and located at ($\text{Re}(\varepsilon) = 0$, $\text{ph}(\varepsilon) = \pi/2$), and $f_0$ is the upper critical frequency that nulls parallel admittance (the shunt path to ground is opened) and locate at ($\text{Re}(\varepsilon) = 0$, $\text{ph}(\varepsilon) = \pi/2$). The other characteristic frequency $f_2$ can be defined as the frequency for which $S_2$ has a minimum and is located at ($\text{Re}(\varepsilon) < 0$, $\text{ph}(\varepsilon) \neq \pi/2$).

![Fig. 4 – Permittivity response ($\text{Re}(\varepsilon)$ and the $\text{ph}(\varepsilon)$) of CSRR unit cell.](image3)

The extraction procedure in this article will show that the calculation of equivalent circuit relies on these three frequencies directly, as it will be illustrated in next section.

3. DESCRIPTION OF THE TECHNIQUE

At resonance, CSRRs have small electrical dimensions and hence the unit cell can be depicted by agency of lumped-element equivalent circuits. The technique explains that the effect of negative permittivity band has an essential role to extracting the equivalent circuit model of a CSRR-loaded transmission line.

From circuit in Fig. 2, the inductance and capacitance of the host microstrip line, $L$ and $C$ are computed as the per-unit-cell inductance and capacitance of the host transmission line [23]
explained further on. From Fig. 4, \((f_s = 1/2\pi (L_c + C_c) )\) can be determined through full wave electromagnetic simulation directly, or it can be extracted as representing the mid-point of the interval of negative permittivity band \( (f_s = f_h + (f_h - f_s) / 2) \) as depict in Fig. 5, as well as the frequency \( f_s \) locate at \((\text{Re}(\varepsilon) < 0 \) and the phase \( \neq \pi/2 \)). This is considered an advantage given by the characteristic of negative permittivity band.

The expression for \( f_s \) depends on two parameters \((L_c, C_c)\), which are not yet calculated, therefore an additional condition is needed.

From the circuit of Fig. 2, the transmission matrices of the unit cell can be written \([24, pp. 177 – 193]\).

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix} =
\begin{bmatrix}
\frac{1 - \omega^2 L_c (C + C_c)}{2} & \frac{1 - \omega^2 L_c (C + C_c)}{2} \\
\frac{1 - \omega^2 L_c (C + C_c)}{j\omega C_c(1 - \omega^2 L_c)} & \frac{1 - \omega^2 L_c (C + C_c)}{j\omega C_c(1 - \omega^2 L_c)}
\end{bmatrix}.
\]

(3)

The dispersion relation of CSRR unit cell can be calculated as [7]

\[
\cos(\beta L) = \frac{A + D}{2} = 1 + \frac{L}{\frac{1}{C} + \frac{1}{C_c}},
\]

(5)

where \( \omega \) is the angular frequency and where the phase is \( \pi/2 \) (lower frequency, \( f_s \)) (see Figs. 4 and 5). The frequency \( f_s \) is obtained directly as the upper frequency at which \( \text{Re}(\varepsilon) = 0 \) and the phase is equal to \( \pi/2 \). Therefore, it is not required a representation of reflection coefficient \( (S_{11}) \) curve on smith chart to find \( f_s \) as in traditional method [19 – 21]. This is the second advantage offered by features of negative permittivity band.

Now from \( f_s, f_h, f_s, \) and (5) the composite the parallel impedance \((L_c, C_c)\) can be determined. In order to demonstrate the authenticity of the proposed technique, the procedure described above is used to obtain electrical parameters of the CSRR unit cell which is designed and simulated in Fig. 2 and the results are synthesized in Table 1.

<table>
<thead>
<tr>
<th>Lumped elements</th>
<th>( L ) (nH)</th>
<th>( C ) (pF)</th>
<th>( C_c ) (pF)</th>
<th>( L_c ) (nH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>2.667</td>
<td>0.6727</td>
<td>1.0625</td>
<td>2.4661</td>
</tr>
</tbody>
</table>

From the elements in Table 1, zero transmission in frequency response of the structure can be obtained by electrical simulation (using MATLAB) as well as electromagnetic simulation as depicted in Fig. 6. Good agreement has been achieved between electrical simulation and electromagnetic one, which validates the structure model and the proposed procedure for parameter extraction.

Further investigation is shown in Fig. 7, which depicts the electromagnetic simulation of the reflection coefficient \( (S_{11}) \) on Smith chart. It can be noted that the intersection of \( S_{11} \) curve with unit resistance circle provides finding \( f_0 \) (2.613 GHz) (as in traditional method), in agreement with \( f_0 \) (2.65 GHz) selected from Figs. 4 and 5. Hence, further confirm of the validity of the proposed procedure is obtained.

### 4. CSRR UNIT CELL WITH DIFFERENT LENGTHS OF MICROSTRIP LINE

Furthermore, to confirm the validity of the proposed procedure in Fig. 6, the CSRR loaded with different non periodic lengths of microstrip lines have been designed and simulated. In that sense, \( L \) and \( C \) per-unit-cell of the host microstrip line in the equivalent circuit model are different in each unit cell and hence the property of the material will be effected \((\text{Re}(\varepsilon) < 0 \) and \( \phi(\varepsilon) \)). Three units are utilized with different non periodic lengths which are 15, 18 and 22 mm.

The simulation results are shown in Figs. 8 and 9. The interval band for which \( \text{Re}(\varepsilon) < 0 \) is shifted to lower frequencies and decreases accordingly to the variation of
the host length as depicted in Fig. 8. The phases of ε, \( \phi(\varepsilon) \) with locations of the critical frequencies for different lengths are depicted in Fig. 9. It is very important to note that the critical frequencies which are represented by lower and upper frequencies for each negative band are shifted and no longer have the same description as before. Since that, the proposed procedure for extracting the equivalent circuit is dependent on the location points of the critical frequencies that limit the interval of negative permittivity band, so it is necessary to reanalysis the proposed procedure to become more comprehensive.

**Fig. 8** – Permittivity response of CSRR loaded different lengths (11, 15, 18 and 22 mm) with (20.2 %, 14.76 %, 13.1 % and 11.67 %) fractional bandwidth for each length respectively.

**Fig. 9** – Permittivity phase response of CSRR loaded different lengths which are (11, 15, 18 and 22 mm).

Figure 10 depict the real part of ε, \( \text{Re}(\varepsilon) \) and its phase, \( \text{ph}(\varepsilon) \) (for CSRR loaded with 22 mm microstrip line length), as well as three location points of frequencies \( f_s, f_{sh} \) and \( f_0 \). Due to the shift which appears in the negative permittivity band, \( f_0 (\text{Re}(\varepsilon) > 0, \text{ph}(\varepsilon) = 0) \) which represent the upper frequency has no longer the same description as in Figs. 4 and 5, but it’s replaced by \( f_{sh} \), which represents the resulting effect of shifting in the negative permittivity band, however \( f_s \) still represents the frequency that will used in the proposed procedure to determine the parallel impedance and consequentially \( L_c \) and \( C_c \).

**Fig. 10** – Permittivity response \( (\text{Re}(\varepsilon) \text{ and the ph}(\varepsilon)) \) of CSRR loaded transmission line of 22 mm length.

The transmission frequency response of the CSRR loaded with these different lengths of microstrip lines have been simulated. The results are given in Fig. 11, which depict comparison between electrical simulation results and electromagnetic simulation ones. It can be noted that the electrical results are shifted corresponding to the shifting in negative permittivity band due to variation of the host length. Nevertheless, simulated resonance transmission frequency of rejection band (dip of \( S_{21} \)) remain compatible with the equivalent circuit model for each length of microstrip line, and hence further confirm of the validity of the proposed procedure.

**Fig. 11** – Magnitude of the transmission coefficient with respect the frequency for CSRR loaded with different lengths of the transmission line.

The electrical lumped parameters calculated are presented in Table 2. It can be noted that the maximum effective change is specified by \( L_c \) and \( C_c \), which increases the capacitive effect of CSRR impedance that is composed of \( L_c \) and \( C_c \). It is interesting to note the increment in both \( C \) and \( C_c \) values, which leads to decrease of values for critical frequencies \( (f_s, f_{sh}) \) (see Fig. 8), hence, negative permittivity band will shift to lower frequencies. This confirms that the equivalent circuit parameters are inherent to the negative permittivity.

**Table 2**

Extracted parameters of CSRR loaded with different lengths of the transmission line

<table>
<thead>
<tr>
<th>L of TL/Elec.</th>
<th>( L ) [nH]</th>
<th>( C ) [pF]</th>
<th>( C_c ) [pF]</th>
<th>( L_c ) [nH]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = 15 ) mm</td>
<td>4.0427</td>
<td>0.9173</td>
<td>3.0577</td>
<td>1.1193</td>
</tr>
<tr>
<td>( L = 18 ) mm</td>
<td>5.1433</td>
<td>1.1007</td>
<td>5.924</td>
<td>0.6302</td>
</tr>
<tr>
<td>( L = 22 ) mm</td>
<td>6.6838</td>
<td>1.3454</td>
<td>15.377</td>
<td>0.2266</td>
</tr>
</tbody>
</table>

**Fig. 12** – Electromagnetic simulated of reflection coefficient \( (S_{21}) \) on a Smith chart for CSRR loaded with different lengths of microstrip transmission line (11, 15, 18 and 22 mm).
Another investigation is shown in Fig. 12, which depicts the electromagnetic simulation of reflection coefficient ($S_{11}$) on a Smith chart for the CSRR unit cell, which has different lengths of transmission line. It can be noted that the upper frequencies ($f_0$) provided for each length of the host line (2.613, 2.448, 2.355 and 2.528 GHz) are close and in agreement with corresponding upper frequencies (2.65, 2.4, 2.47 GHz) that were extracted in Fig. 9, so a further evidence is added to the validity of the proposed procedure.

5. CONCLUSIONS

In this work, a new study relying on the negative electrical permittivity band to extract the electrical parameters of the equivalent circuit of complementary split ring resonators (CSRRs) loaded transmission line is proposed. CSRR coupled to microstrip transmission line has been suggested and the behavior of related frequency band of negative permittivity has been investigated, in order to provide the frequencies that are adopted in the calculation of electrical parameters of the equivalent circuit. The equivalent circuit has been simulated and a good agreement between the results obtained for resonant frequency derived from the simulation results and equivalent circuit model is obtained. Simulation results of transmission frequency rejection band remain compatible with results of equivalent circuit model when different lengths of transmission line are applied and hence, further validity of equivalent circuit is confirmed. The increment in host line capacitance ($C$) leads to an increase of $C_{\text{v}}$ value, hence shifting the interval of negative permittivity band to lower frequencies. This confirms that equivalent circuit parameters are influencing the negative permittivity. Moreover, perfections to the proposed procedure can be obtained by verifying its validity when the proposed procedure is applied on negative permeability line structure, as a suggestion for future work.

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